Assessing the Systemic Risk in the Brazilian Interbank Market

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Abstract

In this paper, we propose a methodology to measure systemic risk that stems from financial institutions (FIs) interconnected in interbank markets. We show that this framework is useful to identify systemically important FIs, and to describe the group formed by them. This methodology can be used to perform stress tests using additional information from FIs default probabilities and their correlation structure. We present how to implement this methodology and apply it to the Brazilian case. We also evaluate the effects of the recent global crisis on the interbank market.

Keywords: Systemic Risk; Financial Stability; Interbank Market; Contagion; Macroprudential; Networks.

JEL Classification: G21, G23, C63, L14.

1 Introduction

The occurrence of international financial crises in recent years have highlighted the need of understanding and assessing systemic risk, which allows its mitigation and crises prevention. Besides the identification of mechanisms that may lead a financial system to a systemic crisis, it is necessary to identify the financial institutions (FIs) that may play a key role on a crisis onset. Furthermore, it is necessary to have a tool for assessment of the condition of a financial system at a given time: is it next to a crisis? Is it possible to intervene to mitigate this risk and assure the financial stability? How to do it optimally, using the minimum resources possible to get the desired effect?

Systemic crises usually begin in a single FI or in a little group of them and spread to a larger portion of the financial system, eventually affecting the real sector. Besides the surveillance of individual FIs, it is necessary to identify contagion mechanisms and define measures to be taken ir order to mitigate the effects it provokes. There are four basic contagion mechanisms: 1) Risk concentration: a great number of FIs is exposed to a common risk factor; 2) Balance sheet contagions, by the write off of assets held by counterparties; 3) Price-mediate contagion, due to fire sales, and 4) The occurrence of illiquidity spirals, due to margin calls or short term liabilities. The literature has been concerned with the possibility that the way FIs form networks when relating one with others is relevant to the contagion process. See, for instance, Boss et al. (2004), Furfine (2003), Wells (2004), Inaoka et al. (2004), Soramäki et al. (2007), and Boss et al. (2008). This paper contributes with this literature's stream focusing on the balance sheet contagion process.

The contribution of this paper is fourfold. First, we adapt the methodology presented by Battiston et al. (2012) and propose a methodology to measure systemic risk that stems from FIs interconnected in interbank markets. Second, we show that this framework is useful to identify systemically important FIs, and to describe the group formed by them. Third, we also show how to perform stress tests using additional information from FIs default probabilities and their correlation structure. Fourth, we present how to implement this methodology and apply it to the Brazilian case. We also evaluate the effects of the recent global crisis on the interbank market.

The systemic risk is estimated from impact measurements of individual FIs. The impact, or DebtRank, is estimated according to the methodology proposed by Battiston et al. (2012). In their model, the initial shock is a initial stress applied to one or more FIs. The effects of this initial stress spread along the system by a contagion process. Once the contagion is finished, one sums up each FI stress to get the financial system stress associated with the initial shock. The impact measure is defined as the difference between the system's final stress and the initial stress. Impact varies continually with initial stress variations, which emphasizes its stress nature. Impact can also be interpreted as the potential loss induced by a situation of stress motivated by one or more FIs and is presented as a fraction of the total assets invested in the market. These potential losses are used to evaluate the systemic risk associated with individual FIs and, hence, to identify the systemically important financial institutions.

To measure the financial system's stress, one approach is to define it as the sum of the impacts computed for each single FI's default. This has an implied assumption that every FI has the same default probability, equal to one. However, FIs have different risk profiles and, consequently, different default probabilities. We aggregate this information in the model and assess systemic risk as the financial system's expected impact over a 1-year-horizon. We use monthly accounting data and the Merton's structural model (see Merton (1974)) to estimate the default probabilities, which are necessary to estimate the expected impact. To the best of our knowledge, this is the first paper that evaluates systemic risk combining default probabilities and potential losses given default computed using network analysis methodologies.

We use the DebtRank model's property that a initial shock may be applied to one or more FIs to perform a stress test. This paper presents a methodology for building stress scenarios which consists in identifying a set of FIs that have default probabilities highly correlated. We assume this set of FIs default simultaneously and compute the impact resulting from this shock.

Interbank Markets play an essential role in a well-functioning integrated financial system through the provision of liquidity among banks. The FIs lend or borrow money among themselves and make commitments taking into account the repayments at the due dates. If an FI fails in the repayment of its loans, its creditors may have trouble in honoring their debts, propagating the effects of the original failure to other institutions, in a contagion process. Problems affecting one institution may spread to other ones and even to institutions across international borders. Given the importance of this market, we identified the FIs that are sources of systemic risk in the Brazilian interbank market.

The whole framework is concerned with solvency issues and do not take into account the institutions' liquidity, modeling, from the four channels of contagion listed above, the balance sheet contagion channel only. In spite of this restriction, it has the advantage of measuring the stress of the entire financial system or the one related to individual institutions from the point-of-view of the contagion channel modeled. This gives room to analysis of the sources of stress within the financial system, the identification of systemically important financial institutions along time, the characterization of these institutions and of the group they form. The proposed methodology can be used as an auxiliary tool for monitoring the financial system, indicating the overall and local stress. Given this information, a central bank would have more data to properly define stabilizing actions to be taken.

The analysis of the individual FIs' impact measurements shows that high impact FIs may have neither a large size nor a large participation on the assets invested in the interbank market. Conversely, there are large FIs with a low impact, being this impact lower than that of some smaller size FIs. Similarly, a greater FI fragility is not necessarily associated with a higher impact measurement[1].

We analyzed the relationship between impact and FI conditions to identify the main factors that contribute to a high impact measurement, finding that it is important: 1) the FI's interbank market total liabilities' share; 2) the FI creditors' conditions: the FI which creditors both own a large share of the market's assets and have invested in that FI an amount larger than their capital buffer, and 3) that the FI have creditors that amplify losses, that is, creditors in the situation described in 2) that possess interbank liabilities greater than their assets in that market.

We find that, in the period of analysis (July 2011 to June 2012), the group formed by the highest impact FIs is stable, with a low turnover. The expected impact computed for the financial system remain stable in this period, besides, some small or micro-sized FIs contribute significantly for the financial system expected impact due to their greater default probabilities.

To perform the stress test, we use the methodology proposed in the paper to identify a group of 10 medium and small-size FIs and simulate their joint default, getting an impact measure comparable to the largest FI's one. This FIs' group own 5% of aggregated tier 1 capital and 6.7% of interbank market assets, producing a 15.1% impact, while the highest impact FI in the period own 13.6% of interbank market assets and produces a 14.5% impact. This group is an example of a loss amplifying FIs' group[2]: its liabilities' share in the interbank market is 19.1%, contrasting with its 6.7% assets' share. The existence of such groups draws attention to the possibility that major losses may be originated in a group of individually low-relevance FIs.

In the following section, we present a literature review on contagion and systemic risk models, emphasizing the network based ones. The third section reviews the DebtRank and Merton's structural model methodologies used in the computations. The fourth section brings information on the sources and meaning of data used in this paper. We also perform a first network analysis there. The fifth section presents analysis and results, and the last section concludes.

2 Literature

Contagion is a key factor for systemic risk, hence there is a growing literature on contagion between FIs, addressing its theoretical foundations (e.g., Rochet and Tirole (1996) and Allen and Gale (2001)), the identification of contagion mechanisms for several markets (e.g. Degryse and Nguyen (2007), Lehar (2005), Elsinger et al. (2006) and Mistrulli (2011)), the proposition of models (e.g. Elsinger et al. (2006), Iori et al. (2006) and van den End (2009)), empirical tests (e.g. Castiglionesi (2007), Pe et al. (2010) and Hsiao

[1]The FI fragility is measured by its leverage with funds of the interbank market.

[2]The methodology for building stress scenarios we propose does not take impact amplification into account. This stress scenario's characteristic is not intentional.

et al. (2011)) and the investigation of methods of contagion prevention (e.g. Castiglionesi (2007)).

Rochet and Tirole (1996) started the theoretical literature that have been studying the issue of contagion. They develop a model in which decentralized interbank leading is motivated by peer monitoring. They derive prudential rules and looks at the impact of interbank monitoring on the solvency and liquidity ratios of borrowing and lending banks.

Allen and Gale (2001) is a seminal paper that models financial contagion as an equilibrium phenomenon with the result that a small liquidity preference shock in one region can spread by contagion throughout the economy. The authors assess that the possibility of contagion depends strongly on the completeness of the structure of interregional claims, being more robust when complete.

Gridlock equilibrium or coordination failure was studied by Freixas et al. (2000). The authors model systemic risk in an interbank market. Interbank market exposes the system to this failure even if all banks are solvent. They also investigate the ability of the banking system to withstand the insolvency of one bank and whether the closure of one bank generates a chain reaction on the rest of the system. For them, the central bank has a role to play as a "crisis manager".

Empirical work on the interplay between the structure of the interbank market and the risk of contagion has mainly focused on national banking systems. Degryse and Nguyen (2007) employ individual banks time series data on interbank exposures from the Belgian system to investigate the evolution and determinants of contagion risk. They find that a move from a complete structure toward a multiple money-center structure has decreased the risk and impact of contagion that is consistent with a theoretical prediction in Freixas et al. (2000). They also show that the increase in the relative importance of cross-border interbank exposure has lowered local contagion risk as well.

On the other hand, Mistrulli (2007), assessing the risk of contagion for the Italian interbank system and using simulations, concludes that moving from a complete structure to a multiple-money center structure has increased the risk of contagion contradicting the results for the Belgian system reported by Degryse and Nguyen (2007). However, Mistrulli's analysis does not account for the decrease in the degree of internationalization that has actually brought the Italian interbank market to become rather local, contributing to the risk.

Another important empirical evidence is given by Upper and Worms (2004) analyzing German banking system. The authors find that in the absence of a financial safety net (institutional guarantees for saving banks and cooperative banks), there is considerable scope for contagion that could affect a large proportion of the banking system. A financial safety net considerably reduces the danger of contagion.

As Allen and Gale (2004) show, a large number of possibilities exist concerning the relationship between market structure and financial stability. Since these are important arguments and as there are trade-offs between these aspects of the banking system, prudential regulatory intervention and supervision is needed in its various forms.

Contagion has also been measured more broadly by taking into account different shocks. Elsinger et al. (2006) simulate the joint impact of interest shocks, exchange rate shocks and stock market movements on interbank payment flows of Austrian banks. They distinguish between insolvency due to correlated exposures and due to domino effects. The results show that although the probability of contagious default is low compared to the total default probability, there are situations in which up to 75 percent of the defaults are due to contagion.

Furthermore, Elsinger et al. (2006) assess that the simultaneous consideration of correlation and interlinkages does indeed make a difference for the assessment of systemic financial stability. In particular, the probability of systemic events such as the joint breakdown of major institutions is underestimated when correlations between banks are ignored. They show that ignoring interlinkages leads to an underestimation of joint default events. The analysis uncovers substantial differences between banks concerning their impact on others in stress scenarios and clearly identifies institutions with a high systemic impact.

Gropp et al. (2009) use the tail properties of distance to default to study contagion risk and find that both domestic and cross-border contagion is present in Europe. Because the distance to default is derived from equity price data, their approach captures contagion as perceived by banks' equity holders. Market-price-based indicators of bank fragility, such as the distance to default, summarize all available information about a given bank. Hence, this measurement of contagion could be viewed as covering all possible transmission channels of contagion.

A review and criticism of the literature lies in Upper (2007), a survey paper which analyzes computer simulations of contagion due to interbank lending, gathering information on the methodologies they use. He emphasizes that one has to bear in mind the potential bias caused by the very strong assumptions underlying these simulations. Those papers suggest that contagion due to lending in the interbank market is likely to be rare. However, he says, if contagion does take place, the costs to the financial system could be very high, destroying a sizable proportion of the banking system in terms of total assets. He also assesses in particular, that none of the simulations is based on a model that incorporates more than an extremely rudimentary behavior by banks or policymakers. Besides, he stresses the need of taking into account multiple shocks (many simulations usually begin with a shock in a single bank), as in Elsinger et al. (2006), and points to the need of taking into account the relationship between insolvency and illiquidity of banks, for instance an in the Müller (2006)'s simulation.

Another literature stream that has grown is that which uses the network structure of the interbank system to get information on systemic risk. Some papers model the interbank network relationships from payment flows. Inaoka et al. (2004) analyzes the network structure of financial transactions, using data of financial transactions through the Bank of Japan payments' system. They find evidence of a certain degree of robustness within the network and suggest an analysis of its dynamics. Soramäki et al. (2007)

analyze the network topology of the interbank payments transferred between commercial banks over the Fedwire Funds Service (USA), finding that the network properties changed considerably in the immediate aftermath of the events of September 11, 2001. Boss et al. (2008) investigate the relevance of network topology for the stability of payment systems in face of operational shocks, using data of the Austrian large-value payment system ARTIS. They find that network indicators at the node level can have explanatory power but, at the stage of their research, network indicators at the network level seem to be of limited use for stability analysis. Pröpper et al. (2009) present the application of network theory to the Dutch payment system with specific attention to systemic stability. They find this network may be susceptible to directed attacks. Embree and Roberts (2009) describe the daily and intraday network structure of payment activity in the Canadian Large Value Transfer System, finding that there are few systemically important participants, with small variations on their relative importance along time.

Other papers study interbank network credit relationships. Instead of payment flows, they analyze bilateral exposures data. These data are usually unavailable and have to be estimated using some methodology from data extracted from other sources. To analyze the interbank network stability, one applies shocks to one or more FIs. Upper and Worms (2004) analyze the German banking system and find that after a simulated failure of a bank, the financial safety net considerably reduces, but does not eliminate the danger of contagion. Furfine (2003) analyzes the impact of various simulated failure scenarios on the U.S. market, and finds that the contagion risk is economically small. Wells (2004) analyzes the UK market, and finds that the failure of a large UK-owned bank possibly would trigger a multiple bank failure. However, this is very unlikely, once large banks have high credit ratings, which are related to low default probabilities. Battiston et al. (2012) analyze the FED emergency loans dataset and find a group of institutions strongly connected that become systemically important during the subprime crisis peak. They propose a quantitative assessment of systemic risk and find that a systemic default could have been triggered even by small dispersed shocks on the above identified group.

Markose (2012) analyses the financial network structure of the interconnections of international obligations in Global OTC derivatives market to identify the systemically important financial intermediaries, study the nature of contagion propagation and design ways of increasing robustness in the network. She determines the stability of the system using its representation as a network of liabilities expressed as a ratio of tier 1 capital.

This paper assesses the potential loss associated with the default of each FI in the Brazilian Interbank Market, using the DebtRank methodology proposed by Battiston et al. (2012). As in Battiston et al. (2012), the potential loss measure is expressed as a ratio of total assets invested. These potential losses are used to evaluate the systemic risk associated with individual FIs and to identify the systemically important financial institutions. After this, we calculate the expected potential loss for the group of the 40 highest DebtRank FIs, using default probabilities calculated by the Merton's Structural model (see Merton (1974)), for an 1-year horizon. We also use correlations between default probabilities and a minimum spanning tree to identify a set of FIs that could default simultaneously, in order to build a stress scenario for a DebtRank calculation.

3 Methodology

In this paper, we intend to evaluate the systemic risk in the Brazilian Interbank Market. Relationships between FIs that take part in this market with others that do not are not considered in the analysis to be done. This analysis can be readily extended to include other markets, however, they must take into account the institutions' solvency[3].

To perform these analyses, we initially compute the impact individual FIs have on the Financial System, and afterwards, we both calculate the Expected Impact of the entire system and propose a methodology to identify groups of FIs to which it would be desirable to calculate the joint impact. The contribution of this paper is to propose a methodology for computing this expected impact measure and for designing stress scenarios. We also show, using this methodology, that size is not the only important characteristic for a bank to be relevant or considered systemically important.

Impact, in the framework to be proposed in the following, is the financial system potential loss provoked by an initial shock or loss suffered by one or more FIs. Since impact is a potential loss given an initial loss, if one has this initial loss probability, for instance, a default probability, he will be able to calculate the associated expected impact.

We use the Merton's structural model (Merton (1974)) to compute the FIs' default probabilities. These probabilities are computed for each individual FI independently. We neither compute joint default probabilities nor compute conditional probabilities, i.e., the default probability of FIs given the default of a specific FI.

The individual FI's impact is computed using the DebtRank methodology, proposed by Battiston et al. (2012). This methodology considers that the losses due to risk events propagate through the balance sheet of FIs, computing the impact of stress scenarios on the total assets invested by the Financial System participants. These stress scenarios can consider losses due to partial defaults of more than one FI. The impact calculated is expressed as a fraction of the total assets invested by the network players.

The impact calculated can be interpreted as a measure of stress. In models of contagion by defaults, the financial system is assessed as being safe if a default doesn't trigger a default cascade. In this case, the measurements obtained cannot inform how far is the financial system of a default cascade. On the other hand, the impact measured by the DebtRank methodology reports the stress increase in, or shortly before a crisis onset associated with the FIs' capital buffer shrinking that usually occurs in these situations.

Regarding to this, Battiston et al. (2012) claim that DebtRank is a candidate early warning indicator of economic crises. To support this claim, they carry a comparison over time and across institutions of DebtRank, Eigenvector Centrality, Impact Centrality

[3] Impacts are calculated against the tier 1 capital of FIs.

and Default Cascade Impact measures, and find that Default Cascade Impact measure is non-zero only around the minimum of market capitalization, about March 2009, providing a signal only when the situation is already very deteriorated. Eigenvector centrality is poorly sensitive to the increase in impact that institution have on each other and does not detect clearly the peak of the crisis. On the other hand, Impact Centrality detects the peak, but does not deliver the increase of the systemic importance of each institution as this peak approaches. Finally, DebtRank starts increasing much before before the peak of the crisis and detects it clearly.

3.1 Impact Computation

In this section, we present DebtRank methodology, which we use to compute the FIs' impact along this paper. Thus, the terms 'impact' and 'DebtRank' will be taken as synonyms along this paper. For further details on the methodology, refer to Battiston et al. (2012).

The DebtRank methodology is inspired by feedback centrality measures. Feedback centrality refers to all those measures in which the centrality of a node depends recursively on the centrality of its neighbors. One example of this kind of measure is Google's PageRank (see Page et al. (1998)). In PageRank, the quality of a web page inherits part of the quality of the web pages pointing to it. The same reasoning applies to the DebtRank measure: the impact of an FI is a result of the sum of the direct impact it provokes on its creditors with the indirect impact it provokes through them. This last term is calculated from their creditors impact on the system.

The DebtRank methodology, as defined in Battiston et al. (2012), models the interbank market as a directed network, in which the FIs are nodes and the exposures between them are links. These links are represented by a weighted adjacency matrix, which elements A_{ij} are amounts lent by institution i to institution j. The total assets invested by i are given by $A_i = \sum_j A_{ij}$ and the relative economic value of an institution i is given by $\nu_i = A_i / \sum_i A_i$, which is the ratio of i's assets to the total assets invested in the interbank market[4]. Each institution i has capital buffer against shocks, E_i, which is its tier 1 capital. If $E_i \leq \gamma$, the firm defaults, where γ is a positive threshold. If node i defaults, the node j suffers a loss of A_{ji} (the assumption adopted here is that in the short run, there is no losses' recovery). Node j will default, as a consequence, if $A_{ji} > E_j$.

The impact of i on j is $W_{ij} = \min(1, A_{ji}/E_j)$ so that if loss exceeds capital, the impact is 1. The value of the impact of i on its neighbors is $I_i = \sum_j W_{ij}\nu_j$ and measures the fraction of total economic value in the network impacted by i directly. If the institution j defaults, $W_{ij} = 1$ and the direct impact of i on j is its economic relative value ν_i (note that the unit of impact's measurements is the interbank market's total assets' amount). When j does not default, the impact of i on j is proportional to W_{ij}, in order that $W_{ij}\nu_j$ is a stress measure of institution j.

[4] Battiston et al. (2012) suggest that other proxies could be taken for economic value.

To take into account the impact of i on its indirect successors, DebtRank, inspired by the feedback centrality, starts defining the impact centrality of an institution i in terms of the impact of other institutions, as:

$$I_i = \sum_j W_{ij}\nu_j + \beta \sum_j W_{ij} I_j \tag{1}$$

where the second term is the indirect impact of i via its neighbors and the parameter $\beta < 1$ is a dampening factor that could be used to reduce the impact caused by more distant neighbors. In vector notation, (1) can be written as:

$$I = W\nu + \beta W I \tag{2}$$

Solving for I:

$$I = (\mathbf{I} - \beta W)^{-1} W\nu = \sum_{k=0}^{\infty} (\beta^k W^k) W\nu \tag{3}$$

where $\mathbf{I}$ is the identity matrix. This equation converges if the largest eigenvalue of W is smaller than $1/\beta$. When the institution's network has cycles[5], the impact originated by a shock on an institution propagates along its creditors until it reaches again the first node of the cycle. In this case, it would be necessary to recalculate the impact of this institution to take into account the additional shock it received. The recalculations due to the presence of cycles increase the original impacts computed by counting the impact of a node onto another more than once[6]. To avoid the distortion caused by this double-count, Battiston et al. (2012) present an algorithm that allows a node to propagate impact only once. If a node that have already received and propagated impact before receives a new one, it does not propagate it. This algorithm produces the same results as (3) if the network is acyclic and is defined in the following.

Let each institution i'state in the period t be described by two variables:

- $h_i(t) \in [0, 1]$, that is the stress level of i. If $h_i(t) = 0$, i is undistressed, if $h_i(t) = 1$, i is on default.

- $s_i(t) \in U, D, I$, that is a discrete variable which states U, D, I mean i is undistressed, distressed and inactive.

The initial conditions for the simulation are set in $t = 1$. The institutions with initial stress level $h_i(1) = 0$ are undistressed, i.e., $s_i(1) = U$; if $h_i(1) > 0$, the institutions are distressed $s_i(1) = D$; if $s_i(1) = 1$, they are initially on default. The dynamics for each time step, starting from $t = 2$, is given by:

[5]When the network has at least one cycle, there is at least one $k > 0$ for which there exists some i such that $(A)^k_{ii} \neq 0$, where A is the adjacency matrix.

[6]In the case of a clearing system network, this recalculation would have to be done in order to achieve a fixed point in which all the institutions pay their debts or, if not possible, pay all they have(in this case, Eisenberg and Noe (2001) proved that there always exists an unique clearing payment vector for the system, under mild regularity conditions.).

$$h_i(t) = \min\left\{1, h_i(t-1) + \sum_j W_{ji} h_j(t-1)\right\}, \text{ where } j \mid s_j(t-1) = D,$$

$$s_i(t) = \begin{cases} D & \text{if } h_i(t) > 0; s_i(t-1) \neq I \\ I & \text{if } s_i(t-1) = D \\ s_i(t-1) & \text{otherwise} \end{cases} \tag{4}$$

In (4), all the variables $h_i(t)$ are updated from information available from the prior step. After this, the variables $s_i(t)$ are also updated. If $s_i(t-1) = D$, $s_i(t) = I$, preventing institution i of propagating impact to its successors more than once.
After a finite number of steps T, the dynamics stops and the DebtRank can be computed from:

$$DR = \sum_j h_j(T)\nu_j - \sum_j h_j(1)\nu_j \tag{5}$$

DebtRank (or impact) DR measures the additional systemic distress generated by the initial stress scenario set in $t = 1$ and is computed as the difference between the final and the initial systemic stresses.

The losses' propagation process taken into account in DebtRank computation is represented in Figure 1.

<PLACE FIGURE 1 ABOUT HERE>

Suppose that i (financial institution i) defaults. Its creditors, j and k will not have their investments in i repaid, resulting, respectively, in the losses A_{ji} and A_{ki}. To analyze this shock developments for institution j (institution k passes through the same process), we compare the loss A_{ji} with j's capital buffer, E_j. If W_{ij} $(= A_{ji}/E_j) \geq 1$, j has not the capital needed to absorb the loss and defaults (goes insolvent). This default has implications both on the assets side and on the liabilities sides of the balance sheet. For the analysis in the following, it is assumed that the liquidation process occurs in the long term (months or years), while the loss propagation process is finished within a few days, if much. We also assume that, once the financial institution defaults, the liquidation process begins immediately, giving no room to fire sales or similar attempts to raise funds.

Due to the liquidation process, in the short term, assets and liabilities are frozen[7], therefore, j's assets will be frozen, becoming unavailable to j and to the entire financial

[7]This assumption has already been used by Elsinger et al. (2006), who simulate "short term" and

system. On the liabilities side, j, in default, will not do any payment to its creditors. The j assets' freezing is represented in the DebtRank model as the withdrawal from the financial system of the economic value of j, while its default is represented as the impact transmission from j to its creditors. So, the overall institution's impact is represented as the sum of the direct impacts (losses) generated into the assets side and the indirect impact due to its default in the liabilities side, which transmit to its creditors the same effect received from its debtor initially in default. This is the process that takes place when j defaults.

Supposing that j do not defaults after i's default, i.e., if $W_{ij} < 1$, j's direct and indirect impacts are multiplied by W_{ij}, the impact factor of i onto j. It is this definition in the impact computation, the treatment of 'partial' defaults, that makes the impact measure be representative of the financial system stress, given that its values vary continually from zero to the full default figure[8].

3.2 Default Probabilities

To compute the default probability of FIs, we use Merton's structural model (see Merton (1974)), which is the Contingent Claim Analysis applied to the measurement and analysis of credit risk[9]. Under this framework, the institution's balance sheet is separated in its main components: assets, liabilities and capital (equity). The basic idea of this framework is to model the institution's capital as a call option, with strike price equal to its obligations' promised payments and time to maturity T. In the event of default, equity holders receive nothing, because assets aren't enough for paying debts. If the institution doesn't default, equity holders receive the difference between the value of assets and debt. This option's payoff to be received by equity holders is $\max[A - DB, 0]$, where A is the assets' value and DB is the promised payments total. According to Black and Scholes (1973), the value to be received by the equity holders is given by:

$$E = A\ N(d_1) - DB\ e^{-rT}\ N(d_2) \tag{6}$$

where A is the implied value of the asset, DB is the debt to be paid, r is the risk-free interest rate, T is the maturity of the option, $N(\cdot)$ is the cumulative normal distribution

"long term" default cascades. In a "short term" default cascade, it is considered that the defaulting banks just do not pay any of their creditors, as the default cascade occurs before the defaulting banks' liquidation processes are finished (in this condition, the defaulting banks' payments are frozen), whereas the "long term" refers to the condition in which the default cascade occurs after the liquidation processes of defaulting banks are concluded. In these cases, it is assumed that their creditors receive payments proportional to the debts' values.

[8]For instance, if $W_{ij} = 0.9$, j is almost in default, which would happen if $W_{ij} = 1$. In spite of this, not being in default, j makes its payments in full and the financial system seems to be safe, if considered from a default cascade point-of-view. From a stress point-of-view, the system presents potential losses close to a default's.

[9]Contingent Claim Analysis is a generalization of the Black and Scholes (1973) option pricing theory.

and d_1 and d_2 are defined as:

$$d_1 = \frac{\ln(\frac{A}{DB}) + (r + \frac{\sigma_A^2}{2})T}{\sigma_A \sqrt{T}} \tag{7}$$

$$d_2 = \frac{\ln(\frac{A}{DB}) + (r - \frac{\sigma_A^2}{2})T}{\sigma_A \sqrt{T}} \tag{8}$$

From Black and Scholes (1973) it is possible to obtain the equation that relates assets and equity volatilities:

$$\sigma_E = N(d_1)\frac{V}{E}\sigma_A \tag{9}$$

With information on the market value and on the volatility of equity, and on the book value of liabilities, it is possible to estimate the implied value for A and σ_A by solving the system of equations 6 and 9. The debt to be paid DB is interpreted as a distress barrier. If the value of the institution's implied assets fall below this barrier, it defaults. The distress barrier is given by:

$$DB = STD \ + \ \alpha LTD \tag{10}$$

where STD are the short-term liabilities (maturity $\leq$ 1 year), LTD are the long-term liabilities (maturity above 1 year) and α is a parameter between 0 and 1, assumed by Moody's-KMV, whom we follow, around 0.5[10]. This parameter is a proxy of the share of the long-term liabilities subjected to early redemption in case of stress.

The time to maturity T usually assumed is 1 year and is the horizon for which we compute the default probability. Having defined this, we compute the distance to distress $D2D$, which is the number of standard deviations the asset value is away from the distress barrier:

$$D2D = -d_2 \tag{11}$$

This distance to distress can be used in the computation of a risk-neutral default probability, assuming that the assets have a lognormal distribution:

$$DP = N(-d_2) \tag{12}$$

The default probability is the area below the distress barrier shown in figure 2. For further details, refer to Gray and Malone (2008) and Souto et al. (2009).

[10] According to Souto et al. (2009), Moody's-KMV uses α in the range $0.5 - 0.6$ based on the calibration of their model. This intends to match model and historical probabilities of default.

<PLACE FIGURE 2 ABOUT HERE>

4 Data

The analysis performed in this paper is based on Brazilian domestic interbank market exposures between FIs that participate[11]. The dataset we use includes money-market operations (85% in volume), debentures (13.4%) and repos (1.6%), carried out between conglomerates (groups of FIs that are considered as a single institution) and individual institutions that do not belong to any conglomerate, totalling more than 300 market participants. Exposures between institutions within conglomerates are not considered, only those between institutions from different ones. The domestic interbank market exposures represent about 70% of the total exposures between FIs. The total invested in this market by these institutions varied from R$ 55 billion to R$ 71 billion in the period analyzed, corresponding to about 1.5% of the FIs' total assets, as shown in figure 3.

<PLACE FIGURE 3 ABOUT HERE>

The interbank market is a market that allocate liquidity among FIs, specially in the short term. Figure 4 show the aggregated debt profile on June/2012. Despite 73% of the debt amount matures by 1 year, there are loans maturing on more than 10 years (0.3% in volume - only repos).

<PLACE FIGURE 4 ABOUT HERE>

[11]These FIs comprise: universal banks, commercial banks, investment banks, savings banks, cooperative banks, credit unions, savings and loan associations, consumer finance companies, and brokers and dealers of foreign exchange, government securities, corporate bonds, stocks, and commodities and futures contracts.

In Brazil, domestic interbank market operations must be registered in the CETIP[12]. These operations are unsecured and are subject to early redemption. Taking this into account, we aggregate the open positions between the FIs mentioned above to compose the invested assets matrix for each date in which the analysis is carried. The exposures are taken for the last working day of each month in the period from July 2011 to June 2012. This is done to get exposures data contemporary to the Capital Tier 1 available information.

The institutions are classified by ownership (state-owned, private domestic and foreign institutions) and size (large, medium, small and micro), according to the methodology presented in the Central Bank of Brazil's Financial Stability Report (see BCB (2012)).

Balance sheet data is informed monthly to the Central Bank of Brazil by conglomerates and FIs. These data are used to get the tier 1 capital and other accounting information used along the paper, such as the required in the computations of default probabilities.

4.1 A First Analysis

The FI-level aggregated loan exposures' distribution is represented in a histogram in figure 5. The chart shows the corresponding fitted log-normal distribution. A Jarque-Bera test was performed on the distribution of the sample's exposures' logs, rejecting the null of log-normality (the statistic was 38.19, with a less than 0.1% p-value).

<PLACE FIGURE 5 ABOUT HERE>

The interbank market network's connectivity is given by $p = l/(n(n-1))$, where l is the number of lending relationships and n is the number of FIs in the network. It represents the probability that two given FIs share a lending relationship. Figure 6 shows it is in the range of $1.12 \pm 0.04\%$ along the period of analysis. This shows that the interbank lending network is rather sparse: of the possible lending relationships, only 1.12% occur on average. The lending relationships are not equally distributed among the FIs; each FI has its own relationships, either as a lender or as a borrower. The number of relationships an FI takes part is represented by its out-degree (as lenders) and

[12]CETIP stands for Central de Custódia e de Liquidação Financeira de Títulos. It is a publicly-held company that offers services related to registration, central registration depository, trading and settlement of assets and securities.

in-degree (as borrowers)[13]. We find average degree of 3.01[14]. Figure 7 shows a histogram of the degree distribution of the interbank network on June 2012. This network has 272 FIs, in which 123 are borrowers and 245 are lenders, meaning that the average amount lent by an FI is lower than the average amount borrowed. The histogram presents, for lower degrees, a higher number of lender FIs, and for higher degrees, a higher number of borrower FIs. This means that the most connected FIs are taking part of lending relationships mostly as borrowers (this considers only the number of relationships, not their values). The literature has found that the degrees of both interbank market networks and interbank payment networks have tails distributed as power-laws (see, for instance, Boss et al. (2004), for the Austrian interbank market network, and Soramäki et al. (2007), which analyze the Fedwire network). Despite the histogram shows that, for degrees above 8, the degree distribution could be power-law. This test is not carried formally due to the insufficient number of FIs in the sample.

<PLACE FIGURE 6 ABOUT HERE>

<PLACE FIGURE 7 ABOUT HERE>

Concerning the analysis to be done in the next section, there are two variables that deserve more attention: the shares of FIs which interbank market assets or liabilities exceed their tier 1 capital. The first is important as it represents FIs more exposed to failures of their counterparties, and the second represents the FIs in a most fragile position, which could originate stress or failures in the market. Figure 8 shows that these shares are almost constant along the period of analysis. The more exposed FIs' share is larger than the one of the more fragile (leveraged) FIs. The average ratio of these shares is about

[13]These concepts apply to directed networks, as is the interbank networks' case.

[14]Battiston et al. (2012) find average degree of 12.1, but they consider the 22 core institutions of the US financial system.

that of the numbers of lenders and borrowers. Considering that half of the market's FIs has only one borrower, it is possible that some of them belong to the share of the most exposed ones. Such FIs will default if their borrowers default and are eligible for a more strict surveillance.

<PLACE FIGURE 8 ABOUT HERE>

5 Results and Discussion

The analysis performed in this section are intended to identify groups of FIs that are sources of systemic risk in the Brazilian interbank market, in the period of July 2011 through June 2012. This risk is quantified and the groups' composition is studied along the period of analysis. We compute individual institutions' impact and fragility of for each analysis date, the concentration of impact-related measures and analyze the evolution of this concentration. We also perform analysis of the group of institutions that produce the highest impact in each date, from the point-of-view of the institutions' ownership and size. Another analysis performed is one of the composition's persistence for this group.

Given that impact measures are computed from an institutions's default, we compute the expected impact of the financial system from the default probabilities of the related institutions. The computation gives the financial system's expected impact on an 1-year horizon. Finally, we identify a FIs group with a highly correlated default probability and compute the impact for a simultaneous default of these institutions.

5.1 Individual Impact Measures

Initially, we compute each individual institutions's impact for June/2012. For this calculation, for each institution i, we use (4), considering, as an initial condition, that it is in full stress ($h_i(1) = 1$, i.e., i defaulted). Then, we compute the institution's leverage with interbank market funds, given by $F_i = \sum_j A_{ji}/E_i$, to verify if there are institutions both highly fragile and of a high impact. Table 1 presents statistics for the variables related to this analysis[15], taken for the all FIs' group. Highly fragile institutions which have high

[15]The zero values for the medians of Impact and Leverage occur due to the fact that these values are non-zero only if an FI has borrowed from the interbank market. Of 272 FIs, only 123 have liabilities to

impact would be eligible for a more strict follow-up by banking supervision. Figures 9 and 10 show the results. In these figures, as in the remainder of the paper, the impact's measurements are expressed in interbank market's total assets amount units. In figure 9, x-axis is the Interbank Market Assets Relative Size, in order to allow the visualization of a possible relationship between impact and the institution's participation in the investments in the interbank market. The data points' size is the leverage of the institution with funds from the interbank market, used as a measure of its fragility. The figure shows that the FIs with the highest impact and participation in the interbank market, on the right upper corner of the figure, are not fragile, from the point-of-view of this market. Nevertheless, there are institutions of high impact and high participation in the market that are fragile, with significant leverages. The highest leverages funded by the interbank market are 18.8, 10.3 and 2.4, indicating that these institutions use the interbank markets as a major source of funding. The mean leverage of the top 40 DebtRank institutions, excluding the 18.8 and the 10.3-leverage FIs, is 0.71, and for the top 10 DebtRank group, excluding the same FIs, is 0.34. The 10 largest Total-Assets FIs' leverage is 0.13, which means that their funding is less dependent on the interbank market than the average FI's. Similarly, the top 40 DebtRank FIs' funding depends more on the interbank market than the top 10 DebtRank FIs'.

Variable	Mean	Median	Std Dev	Min	Max
Impact	0.0035	0.0000	0.0142	0.0000	0.1519
Leverage	0.2976	0.0000	1.3810	0.0000	18.7590
Share IB Assets	0.0037	0.0001	0.0149	0.0000	0.1450
Share Total Assets	0.0088	0.0007	0.0303	0.0000	0.1679

Table 1: Statistics of impact measures on June 2012.

In figure 10, the same analysis is performed considering the x-axis as the total assets relative size of the institutions. The figure shows the most fragile institutions, with leverages of 18.8 and 10.3, as being relatively small, with a participation of about 0.3% of total assets[16]. The largest institutions are much less fragile; furthermore, there are large institutions with low impact (see the left lower corner).

<PLACE FIGURE 9 ABOUT HERE>

this market, i.e., less than a half. The low median values of the shares of the interbank market assets and of total assets result from the distributions of these variables.

[16]The FI with leverage 3.5 does not appear in figure 9 because it had no assets in the interbank market on June 2012.

<PLACE FIGURE 10 ABOUT HERE>

We also present in figure 11 the FIs' impact's sum evolution in the period from January 2007 through June 2012. The impact's sum is not the financial system's impact, as explained on the next section, but we can consider it as a proxy for the financial system's stress measure, as it is the sum of the individual FIs' stress. We also present the tier 1 capital's sum and the interbank assets' total for comparison. As seen in section 3.1, an FI impact increases with the sum, for each creditor it has, of the ratio between assets borrowed from him and his tier 1 capital. While the tier 1 capital presents an upward trend until the first half 2011, inclusive, the aggregated interbank market assets increased during the 2008 crisis' onset, experienced a sudden fall on August-November 2008, keeping an almost constant level afterwards. The financial system's stress, as measured by the DebtRank methodology, was higher during the crisis onset, decreasing abruptly on the 2nd half of 2008. This happened due to a 70% interbank market's shrinkage associated with a less pronounced fall of the tier 1 capital[17].

<PLACE FIGURE 11 ABOUT HERE>

5.2 Impact Measures' Concentration

We perform two analyses of the concentration of impact-related variables. In figure 12, we compute the Herfindahl-Hirschman Index for individual institutions' measures. The calculations are made for each date of the period analysis period, for Impact, Capitalization, Interbank Lending, Interbank Borrowing and Total Assets. The indexes computed for each variable show a relatively low concentration, always below 0.12. Impact, Capitalization and Total Assets concentration measures remain nearly constant during the period

[17] On July 2008, the BCB Resolução n° 3.490, August, 29th, 2007 came into effect, modifying banks' capital requirement computation and amplifying its risk coverage. This resulted that, on August 2008, one observes an increase of the tier 1 capital share of the aggregated banks' capital buffer. In the same period, this buffer also increased. See BCB (2008). However, from August to October 2008, the exchange rate depreciated about 30%, reversing temporarily the tier 1 capital growth trend.

of analysis, while Interbank Lending and Interbank Borrowing decrease. We notice that when the top five lending / borrowing institutions are excluded from the calculation, the Interbank Lending / Borrowing concentration indexes become constant along the period (≈ 0.04).

In the case of the lending institutions, some of the top 5 largest lenders reduced significantly their lending along the first halfyear of the period of analysis, possibly due to the worsening of the international scenery associated to the European sovereign debt crisis. Meanwhile, the total assets of the interbank market experienced a reduction, followed by a recovery to a higher level on the following 6 months of the period of analysis.

<PLACE FIGURE 12 ABOUT HERE>

Figure 13 shows the sums of individual institutions' impact, computed for groups of 5, 10, 20 and 40 highest DebtRanks institutions. We notice an increase of the impact measure sums along the first months of analysis in all groups of highest DebtRank measures institutions, followed by a slow-down of this increase. This rise is not concentrated on a particular group of institutions, as can be seen in figure 14. The impact measurements' increase in the period is about 20%. In the same period, the ratio of interbank market assets to tier 1 capital, for the aggregated system, remains almost constant; the share of tier 1 capital aggregated by top impact group remained almost constant for all groups and the shares of interbank market assets aggregated by top impact group increased about 20% for the 5, 10, 20 and 40 top impact FIs' groups, which may explain the impact's increase in the period: the impacts are increasing through the indirect impact channel.

The sum of impacts is not the impact of a group. It is always greater or equal to it, as explained in the following. If two individual institutions are such that the stress of the first of them originates a third institution's default and the stress of the second only stresses the third one, when one computes the group's impact, the additional stress caused to the third institution by the second institution is not counted as it is already on default due to the first institution's stress.

<PLACE FIGURE 13 ABOUT HERE>

In this case, we do not compute the group impact as we intend only to analyze the concentration of impact measures. Figure 13 shows the total impact is increasing along the time, and figure 14 shows that this increase is not isolated into a particular impact group of institutions, being distributed among all groups. The impact sum of the top 5 DebtRank institutions is about 50% of all institutions' impact sum, whereas the 20 highest DebtRank institutions stands for approximately 83% of the overall impact sum.

<PLACE FIGURE 14 ABOUT HERE>

The same analysis is performed dividing the institutions into ownership groups. Figure 15 shows impact sums for all institutions divided into groups of state-owned, private domestic and foreign ones. Whereas state-owned institutions' impact decreases, the private domestic's and foreign's ones increases. The impact of foreign institutions more than double between July and December 2011. This can be explained by their greater exposure to the international market, accompanied by an increase in the interbank market liabilities group share. At the same time, the group of state-owned banks sharply reduced its share in the interbank market liabilities, which contributed to reduce their impact on the financial system.

<PLACE FIGURE 15 ABOUT HERE>

5.3 Highest Individual Impact Group

In this section, we analyze the Top 10 highest DebtRank group. Figure 16 shows that the ownership composition of the group suffers a little change along the time, with the predominance of private domestic institutions. Not only the ownership composition remains stable, but the groups' institutions themselves are rather stable: during the 1-year period, only 14 institutions belonged to the group, of which 5 belonged to it all the time.

Figure 17 presents the group's institution composition, indicating the corresponding type of ownership.

<PLACE FIGURE 16 ABOUT HERE>

<PLACE FIGURE 17 ABOUT HERE>

Regarding the firms' size, the medium-sized institutions are more numerous, followed by the large ones (see figure 18). The group has also one small institution. To explain this groups' composition, one should consider that what causes an institution to have a high impact (to be systemically important) is that it has to produce a high impact (the institution i must have a high W_{ij}) to at least one creditor and, additionally:

- The creditor's economic value must be high, and / or:
- The creditor must have a high impact.

The figure 18 is detailed by figure 19. Is this figure, one notices that there is some persistence in the size of the institution that occupies each position in the ranking along time. Also, it is possible to conclude that systemically important FIs do not need to be large[18].

[18]In fact, on June 2012, the total interbank liabilities of each of 9 from the 10 top DebtRank institutions were of the same magnitude. In this group of 9 institutions, there were medium-sized institutions which liabilities were larger than that of large ones. To explain the mechanics that causes an institution to be of high impact we note that on June 2012, the two highest DebtRank institutions are large and provokes high impact on many creditors on both situations listed above. The third highest DebtRank institution is medium-sized and has high liabilities which provoke high impacts on some creditors of high economic value. The institution ranked tenth is small-sized and, although it does not have a high total liability, it is driven to a single high impact creditor.

<PLACE FIGURE 18 ABOUT HERE>

<PLACE FIGURE 19 ABOUT HERE>

Figure 20 compares the impact of size groups of institutions belonging to the top 10 DebtRank institutions group. It shows that along the period of analysis, large and medium-sized institutions have about the same sum of impacts. As the number of large institutions is lower, the average impact of institutions from this group is higher.

<PLACE FIGURE 20 ABOUT HERE>

5.4 Financial System Impact

The previous impact calculations related a FI's default to its corresponding financial system impact, being an impact given default measure. We now compute the financial system's expected impact over an 1-year horizon. To do so, we use each impact given default measure of the FIs computed before and their corresponding default probabilities in an 1 year horizon. The expected impact of institution i is given by:

$$E[I]_i = DP_i \; DR_i \tag{13}$$

where DR_i is given by (5) and DP_i is obtained from (12). For the whole financial system, the expected impact is given by:

$$E[I] = \sum\nolimits_i DP_i \; DR_i \tag{14}$$

For the computation of (14), we use data from the Top 40 DebtRank institutions. These institutions account for 93% of the sum of individual impacts of the financial system. This group was chosen taking into account the availability of the data needed for the calculation of default probabilities. To compute each of these institutions' default probabilities for a given date (month/year), we use:

A: Total Adjusted Assets, which comprises total assets after netting and reclassification of balance sheet items or groups of items. Netting is performed within the following balance sheet items: repurchase agreements, interbank relations and relations within branches, foreign exchange portfolio and debtors due to litigation. Reclassifications are made within foreign exchange and leasing portfolios;

DB: We compute the distress barrier using $\alpha = 0.5$, consistent with Moody's - KMV *CreditEdge* approach. Due to the unavailability of FIs' total liabilities data, we assume that these liabilities are predominantly short-term with a significant long-term share, from which 50% (α) is early-redeemable: $STD = 0.7$ Total Liabilities and $LTD = 0.3$ Total Liabilities, which results in a DB of 0.85 Total Liabilities. Nevertheless, we perform a robustness check on these assumptions calculating Expected Impact also for $DB = 0.8$, 0.9 and 1.0 Total Liabilities.;

r: the interbank interest rate CDI;

σ_A: the annualized standard deviation of $\log(\frac{A_t}{A_{t-1}})$;

T: We compute the default probabilities in an 1 year horizon using monthly balance sheet data.

We compute the expected impact for all Top 40 DebtRank institutions and for the subgroups of large, medium, small and micro FIs, for the period of Jan/2012 to Jun/2012, and present the results in figure (21). The results are expressed in the same units as the impact calculations'.

<PLACE FIGURE 21 ABOUT HERE>

The standard deviations of the Jun/2012 expected impacts are computed from the default probabilities of these Top 40 DebtRank institutions calculated for the period from

Jan/2010 to Jun/2012. We compute the covariance matrix for these institutions' default probabilities and use it to calculate the expected impact's standard deviation by:

$$\sigma_{E[I]} = \sqrt{DR'\ Cov(PD_i, PD_j) DR} \tag{15}$$

where DR is the column vector of DebtRanks of the Top 40 DebtRank institutions in Jun/2012 and PD_i is the institution i's vector of default probabilities in the period of Jan/2010 to Jun/2012. Expected impacts are computed for $DB = 0.8$, 0.85, 0.9 and 1.0 Total Liabilities, for robustness check. See table 2:

DB/TL	0.80		0.85		0.90		1.00	
2012	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
Jan	0.0126	0.0046	0.0210	0.0057	0.0374	0.0060	0.1179	0.0062
Feb	0.0121	0.0049	0.0209	0.0059	0.0379	0.0062	0.1251	0.0058
Mar	0.0132	0.0051	0.0228	0.0063	0.0416	0.0066	0.1389	0.0060
Apr	0.0118	0.0049	0.0207	0.0059	0.0386	0.0063	0.1359	0.0059
May	0.0128	0.0050	0.0227	0.0060	0.0420	0.0062	0.1423	0.0062
Jun	0.0132	0.0053	0.0232	0.0061	0.0428	0.0064	0.1424	0.0063

Table 2: Expected impact: mean and standard deviation for different values of distress barrier, computed for the Top 40 DR institutions group.

For $DB = 0.85$ Total FI Liabilities, the expected impact's period mean is 2.19% of total interbank assets, with a standard deviation of 0.6%. From table 2 we see that the assumption of higher DBs result in much higher expected impacts, with a maximum of 13.38%, for the case in which one assumes that all the FIs' liabilities are short-term or that α in equation 10 is equal to 1. This expected impact's increase accelerates with the increase of the distress barrier as it corresponds to a distance to default's decrease. Distance to default is the number of standard deviations the assets' value is away from the distress barrier, and is used in the default probability computation using equation 12. The 13.38% expected impact is the cap for expected impacts this model yields, but its actual value depends on the share of the total FI liabilities maturing in 1 year or subjected to early redemption.

We also compute the FI size groups' expected impact and standard deviations and present the means of their monthly amounts along the Jan/2012 to Jun/2012 period in table 3.

We notice that the large institutions as a group present the lowest expected impact due to their reduced default probability. Figure 22 presents the default probability $\times$ impact for the Top 40 DebtRank institutions on June 2012. The dashed curve delimits the region of Expected Impact above 0.001, which have 8 institutions. This region has 5 medium-sized institutions, 2 small and 1 micro.

Group	Mean	Std Dev
All FIs	0.0219	0.0060
Large	0.0002	0.0004
Medium	0.0154	0.0054
Small	0.0037	0.0015
Micro	0.0027	0.0009

Table 3: Expected impact of size groups from the Top 40 DR institutions

<PLACE FIGURE 22 ABOUT HERE>

In the following, we perform a stress test measuring the impact of groups of institutions, instead of one of a single institution. One of the possible ways for the identification of such groups is to select institutions that have default probability above a threshold. For a threshold of 5%, there are 11 institutions from the Top 40 DebtRank group attending to this condition: 6 are medium-sized, 3, small and 2 micro; all are commercial banks, 6 are private domestic and 5 are foreign. Relative to the entire financial system, they account for 5.3% of Tier 1 Capital, 6.4% of interbank market assets, 8.9% of interbank market liabilities and 4.5% of total assets. The group impact is 10.49% of the interbank market assets. Although this is a first step for calculating a group impact, it has the disadvantage of not taking into account that the occurrence of default on the group's institutions may be weakly correlated.

To tackle with this shortcoming, we search for groups of institutions which default probabilities are highly correlated[19]. In such groups, if a factor causes the default probability of one of these institutions to increase, the same occurs to the others in the group. This criterion allows one to identify groups that may have a high default probability in the future even if this probability is not high currently. To identify these groups, we use a Minimum Spanning Tree. Although this technique is concerned with finding the set of edges of a given network that has the shortest total length (see Hillier and Lieberman (1967)), we use it to obtain a graph in which the links between two nodes represent that at least one of them has a default probability highly correlated to the other's. To do this, we associate higher default probability correlations to shorter distances between nodes. After drawing the tree, we use the correlation matrix to identify pairs of institutions highly correlated. We do it for the default probabilities correlation matrix computed for the top

[19]The underlying idea is that if there are common factors rising the default probabilities of a group of institutions, these institutions would have an increased probability of joint default. This would be a proxy for the group's joint default probability.

40 DebtRank institutions and obtain the tree presented in figure 23. The dashed ellipsis shows a group of institutions with highly correlated default probabilities, for which we compute the group impact, which is 15.1% of the interbank total assets. The group has the following characteristics:

- 10 institutions: 6 medium-sized, 4 small, 2 state-owned, 7 private domestic and 1 foreign. 9 are commercial banks and 1 is an investment bank.
- Relative to the entire financial system, the own 5% of tier 1 capital, 6.7% of interbank assets, 19.1% of interbank liabilities and 5.9% of total assets.

Although this group owns 5% of the aggregated tier 1 capital and 6.7% of the interbank market assets, its impact, calculated for June 2012, is much larger, about 15.1% of the interbank market assets. A feature that explains partially this gap is that the group is responsible for about 19.1% of the interbank market liabilities. In the period of July 2011 through June 2012, the highest DebtRank institution owns, on average, 13.6% of the interbank market assets, and has an impact measure of 14.5% interbank market assets, which is of the same order of the group impact computed here. This suggests that the surveillance efforts driven mostly at systemically important institutions may be useful if also driven at the factors related to the default probabilities of these groups of institutions.

<PLACE FIGURE 23 ABOUT HERE>

6 Conclusion

In this paper, we have assessed the systemic risk in the Brazilian Interbank Market using the DebtRank methodology and the Merton's structural model. We use the DebtRank methodology to compute the impacts on the financial system provoked by the failure of each single institution. Given that these impacts are potential losses associated to each institution's default, we compute default probabilities of single institutions to obtain expected losses for the entire system on an 1-year horizon. The default probabilities are computed using the Merton's structural model only for the top 40 DebtRank institutions due to the availability of data. However, this restriction has little impact, as these institutions are responsible for about 93% of the impact sum, on average, during the period of analysis. The time horizon adopted for these default probabilities is 1 year.

The analysis of the expected impact on an 1-year horizon shows that, in the period of analysis, the overall expected impact remains stable, about 2.3% of the interbank market assets, and that the group of medium-sized institutions has the highest expected impact, which happens due to the fact that larger institutions may have higher impact, but have lower default probabilities.

We also find that the FIs with higher default probabilities have little impact, with the exception of one institution, which impact is about 10%. Considering only the impact measures, the group of large FIs have about the same impact as the group of medium-sized ones, considering that this group has more institutions than that group. The analysis of the top 10 DebtRank FIs shows that the composition of this group is stable along time and it is possible that a small institution has a relatively high impact. This is confirmed by the presence of a small institution in the top 10 group in almost all periods analyzed. This supports the claim that size is not the only determinant of which institution is systemically important, but also its position within the network and the condition (fragility, size) of its creditors. This should be taken into account when allocating surveillance efforts in times of stress.

A last finding is that during the analysis period, the foreign institutions experienced a remarkable DebtRank increase, which exceeded that of private domestic ones. This is possibly related to the higher foreign institutions' exposure to banks affected by the ongoing European financial crisis. This finding supports DebtRank as a measure of stress.

This paper contributes with the literature proposing a framework for assessment of systemic risk, taking into account the network relationships of FIs. We move forward with the methodology presented by Battiston et al. (2012), aggregating their DebtRank methodology to default probabilities. This framework is useful to identify systemically important FIs, and to build stress scenarios in which more than one FI undergoes in difficulties. The proposed methodology is used to evaluate the Brazilian interbank market. The results can be applied to identify FIs eligible for a more strict follow-up by banking supervision.

This work should be initially extended to other markets in which FIs have a network relationship, when data is available. The framework presented in this paper focuses on balance sheet contagions and does not take into account contagion mechanisms such as liquidity shortages or contagion through fire sales; however, it is valuable as a measure of stress and for providing insights on factors to be taken into account when assessing systemic risk on a financial network.

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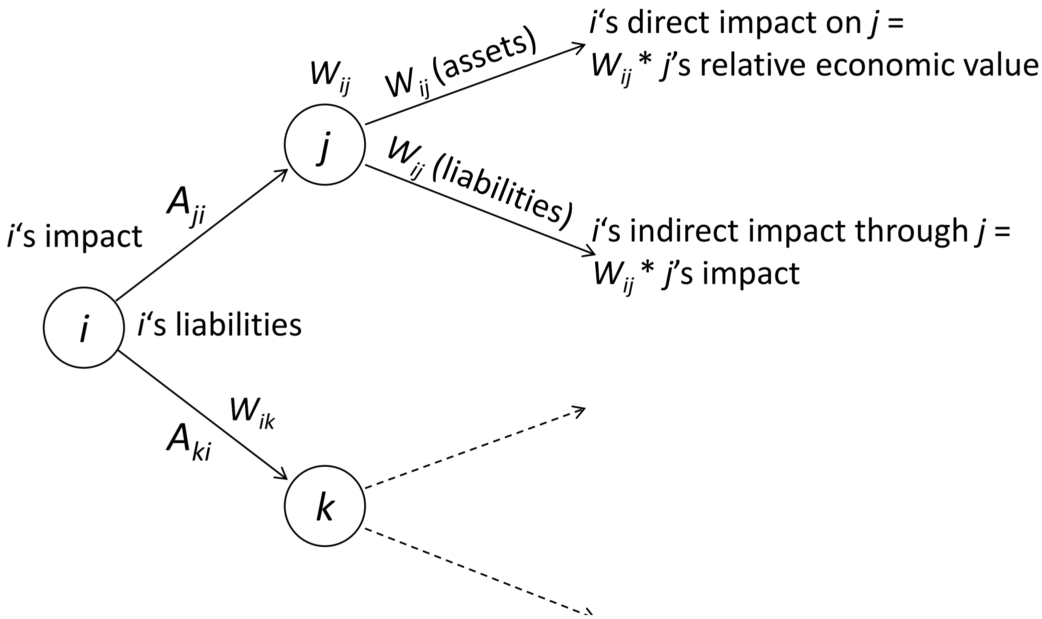

Figure 1: Impact calculation process

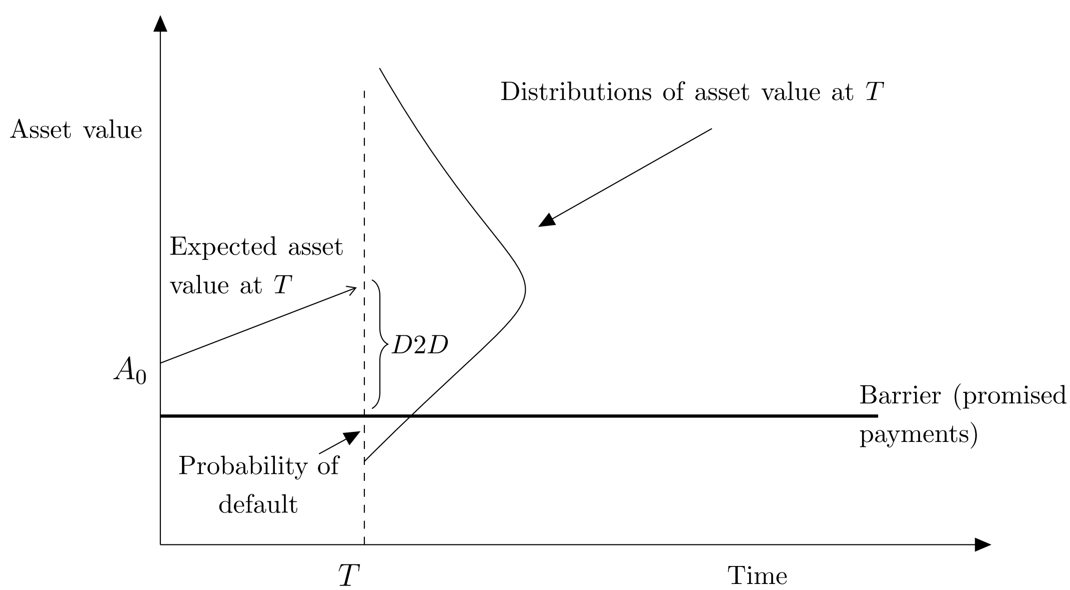

Figure 2: The Merton's structural model

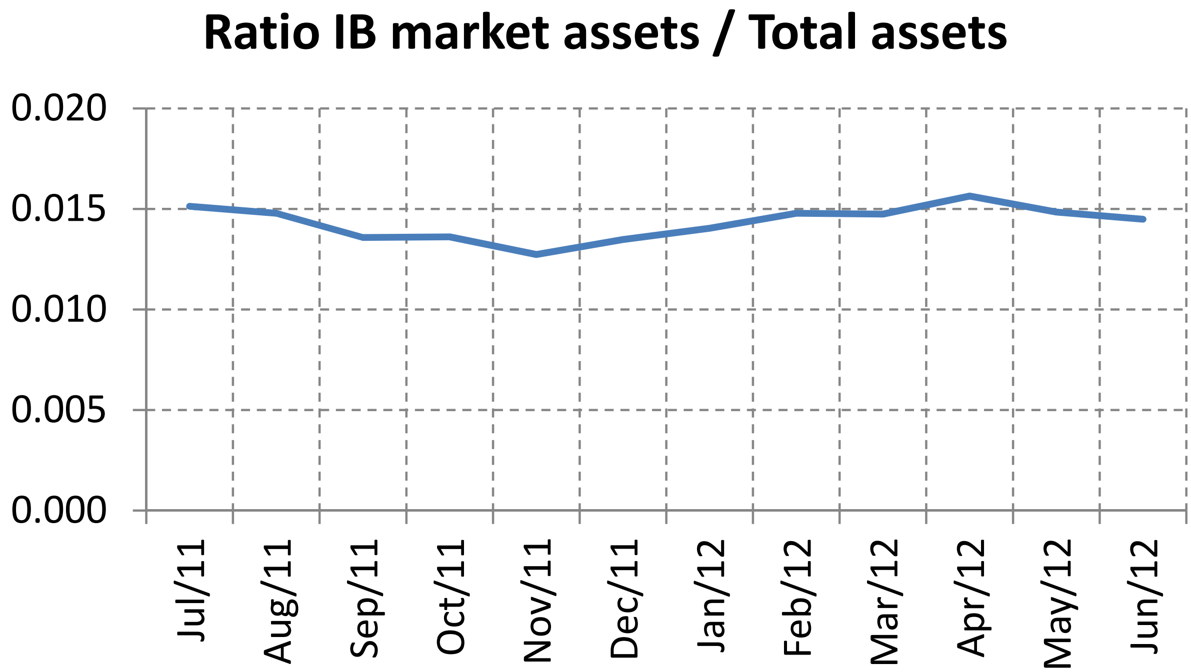

Figure 3: Total FIs' assets' share in the interbank market.

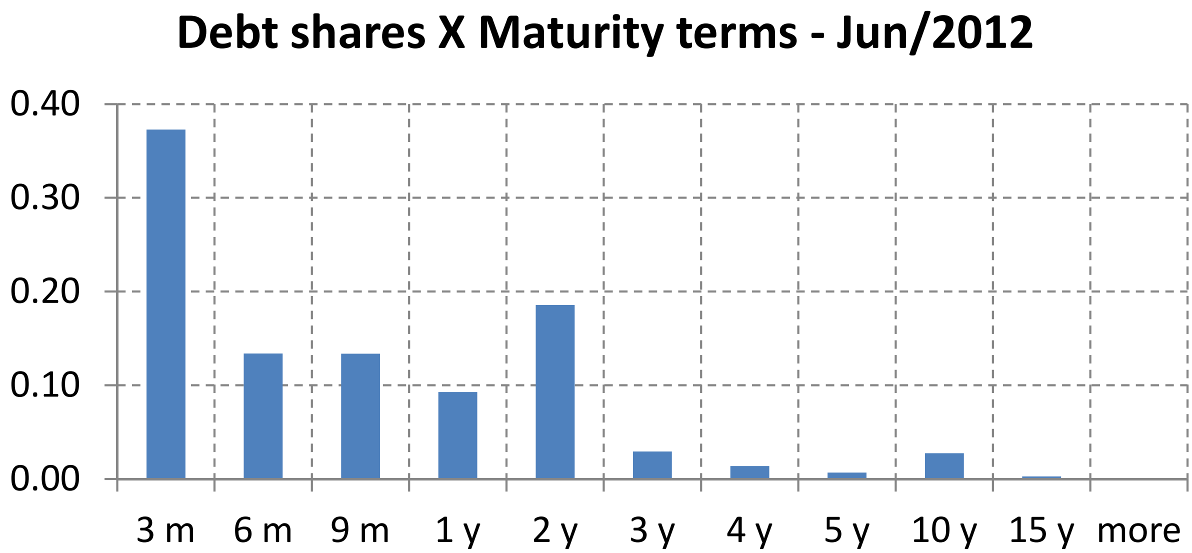

Figure 4: Debt shares × terms to maturity on June/2012.

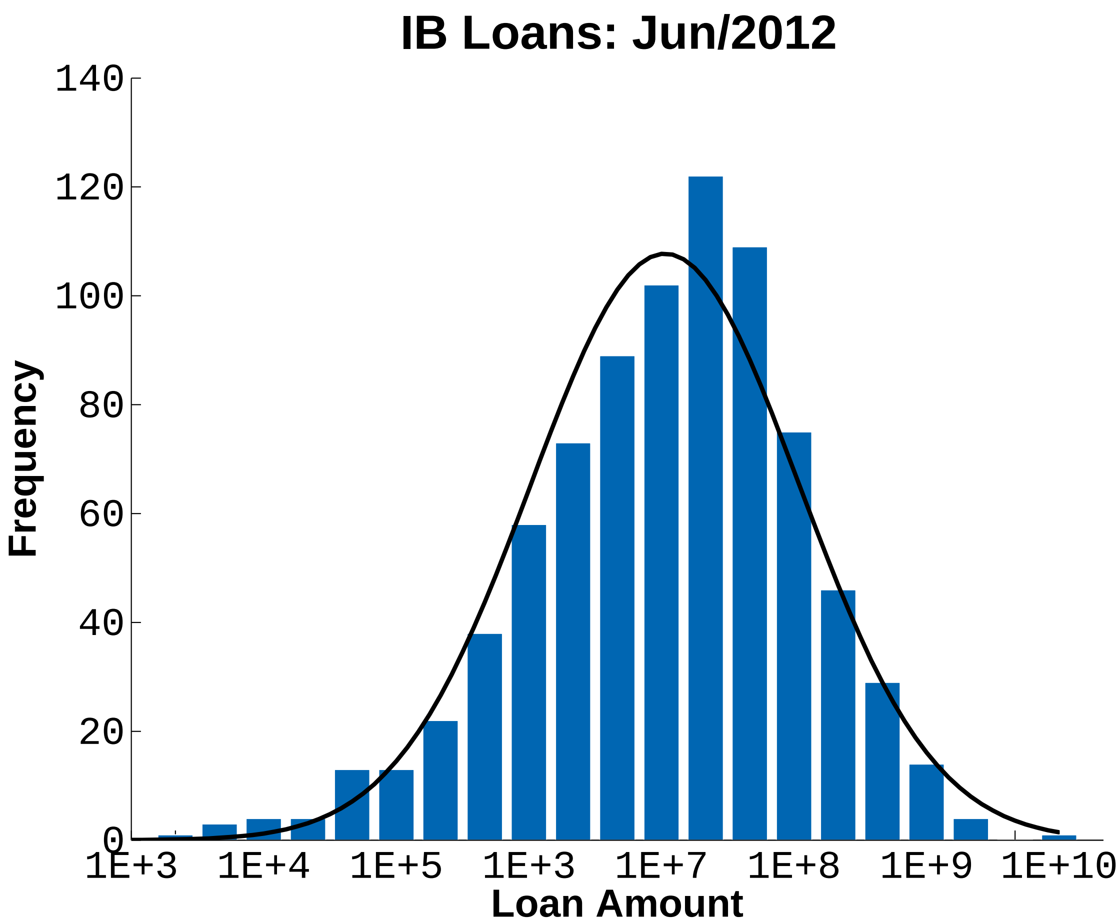

Figure 5: Interbank loans' histogram on June/2012, aggregated by pair lender/borrower.

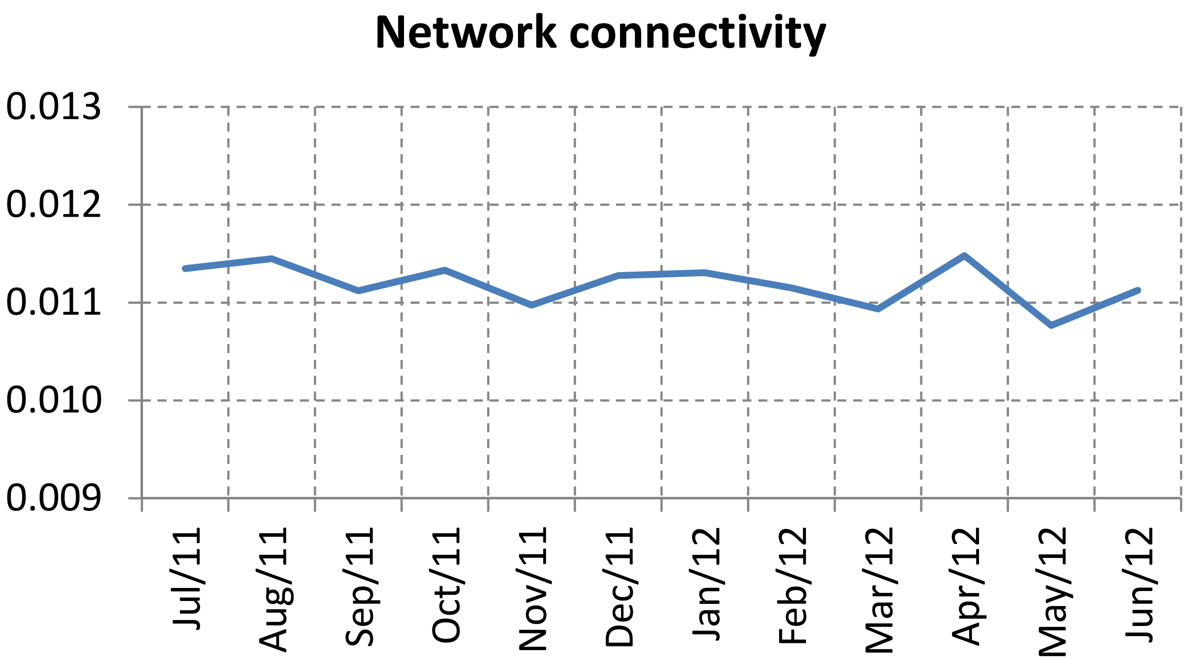

Figure 6: Network connectivity, calculated as the ratio of the number of pairs of FIs connected by a lending relationship to the total of possible FIs' pairs.

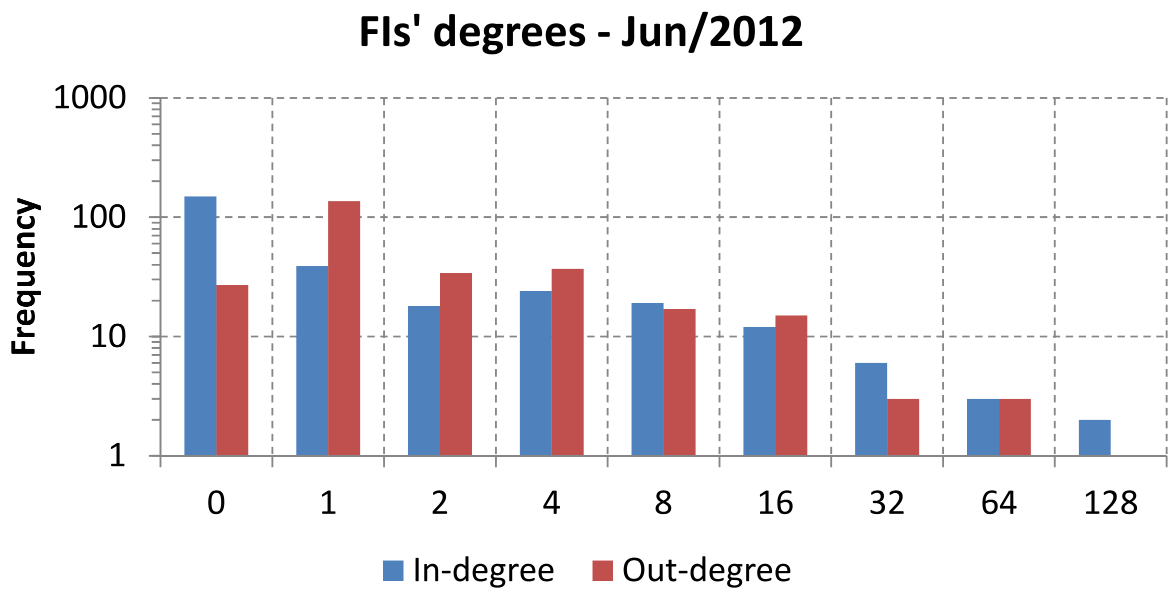

Figure 7: In-degree and out-degree histograms of FIs. The in-degree is the number of lenders an FI has, while the out-degree is the number of its borrowers.

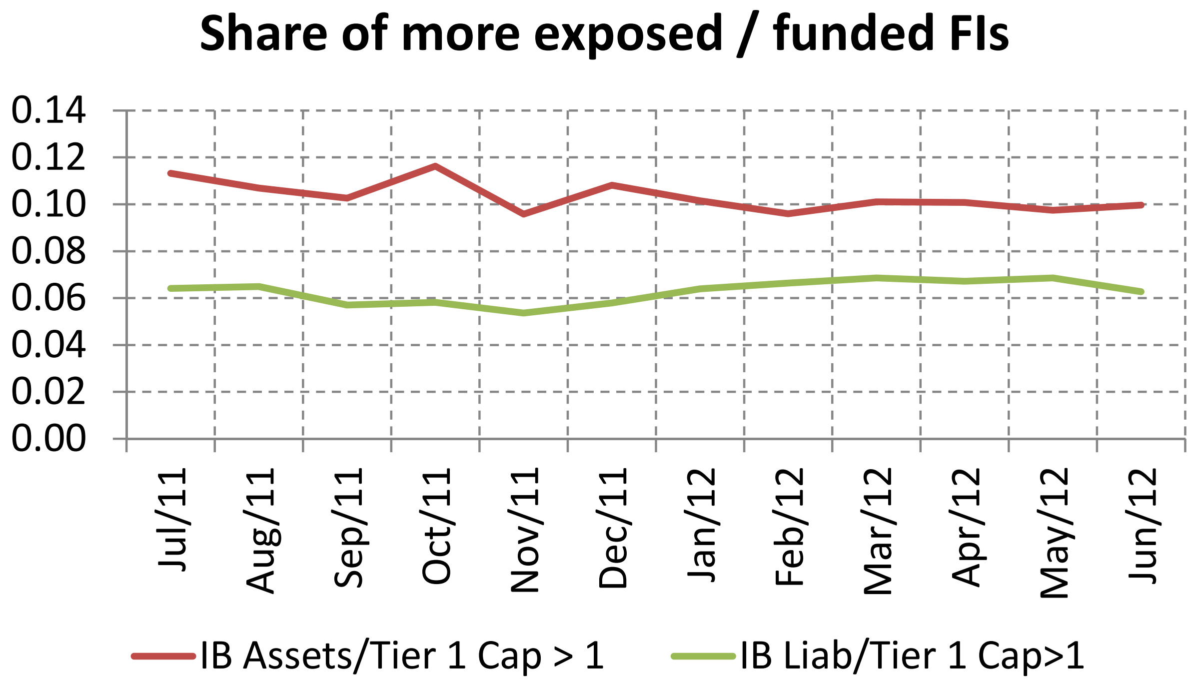

Figure 8: Shares of the total number of FIs that present interbank assets or interbank liabilities to tier 1 capital ratios greater than 1.

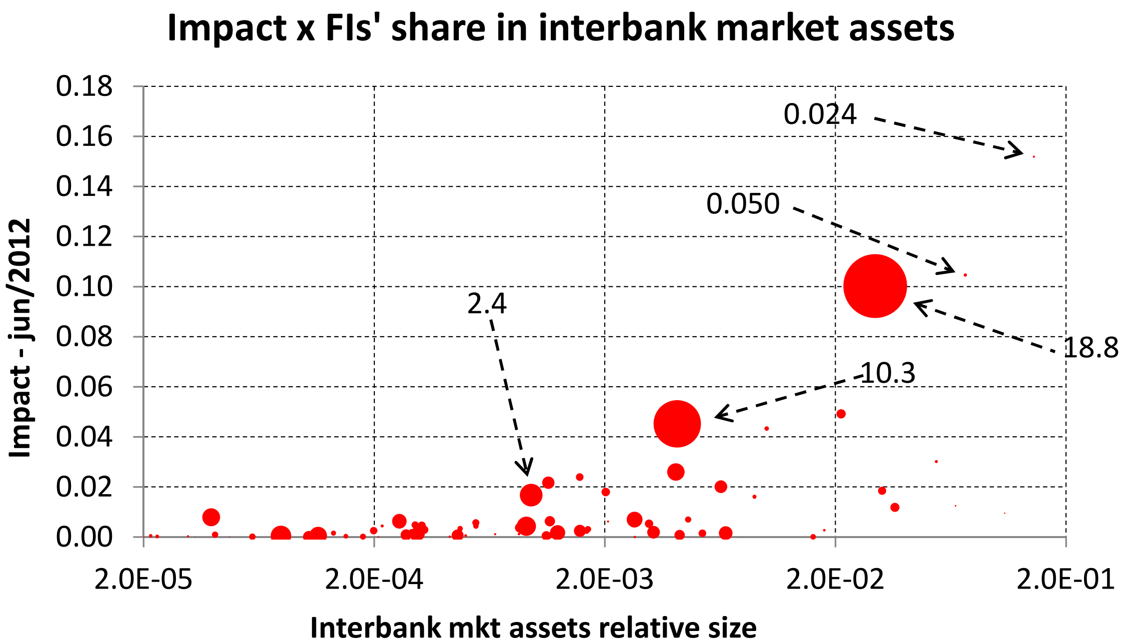

Figure 9: Impact × interbank market share and fragility of FIs. The FI's fragility is measured by its interbank borrowing to tier 1 capital ratio and is indicated by the size of the data point. The FI's interbank market share is computed as the ratio of the FI's interbank lent assets to total interbank assets.

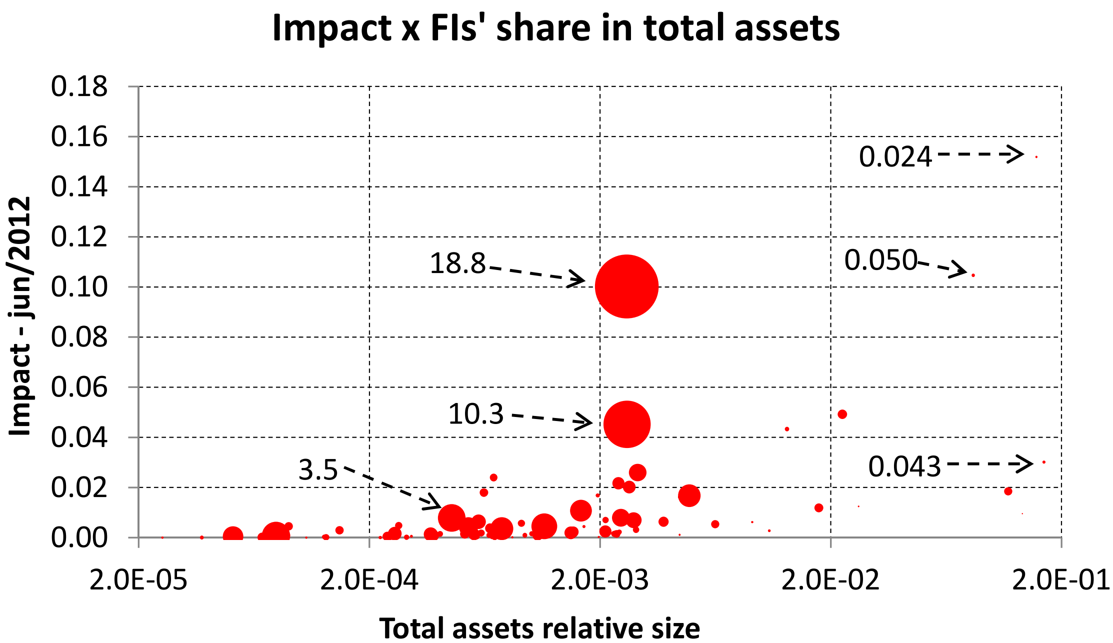

Figure 10: Impact × relative size and fragility of FIs, considering the FI's relative size as the ratio of its total adjusted assets to the sum of these assets for all FIs.

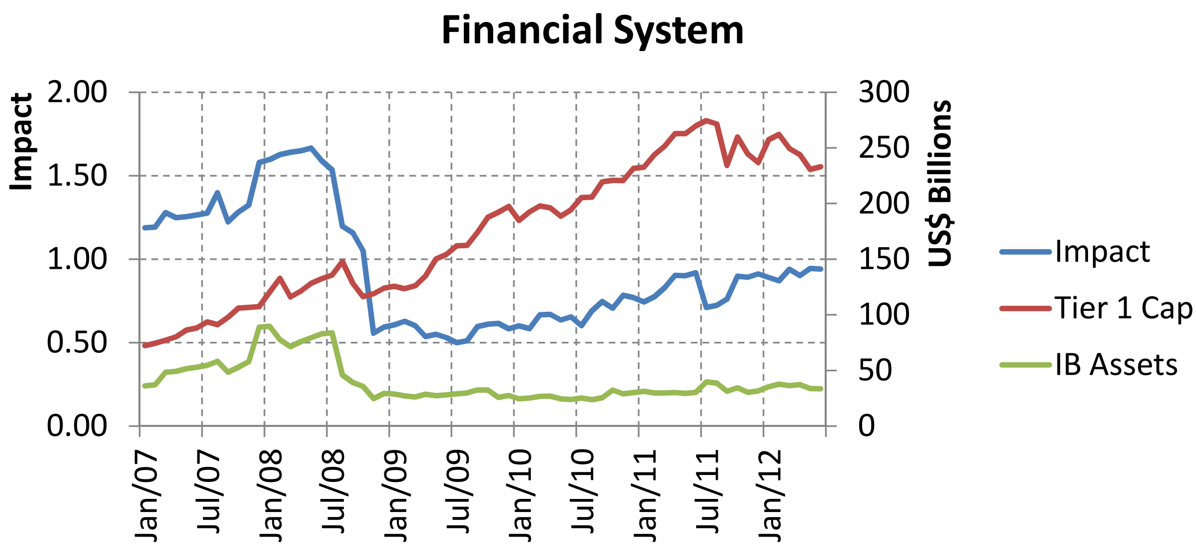

Figure 11: Impact as a measure of stress along the Jan/2007 through Jun/2012 period.

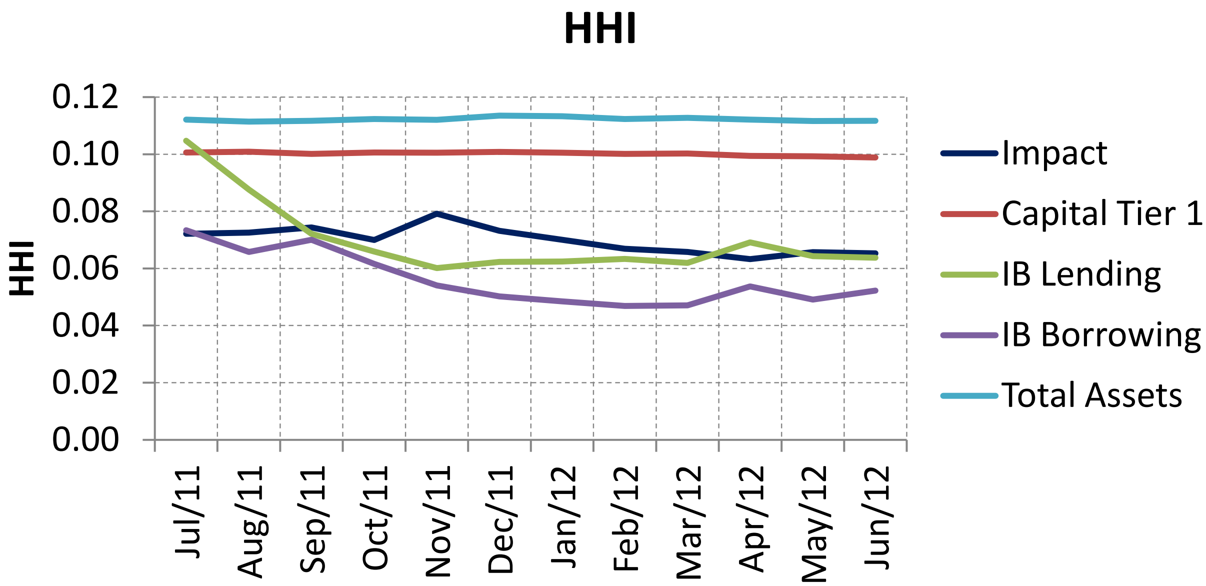

Figure 12: Herfindahl-Hirschman Index for Impact and Impact related measurements. The picture shows, for all of them, low concentration indexes. Interbank lending and borrowing decrease until the end of 2011, while the others remain approximately constant during all the time.

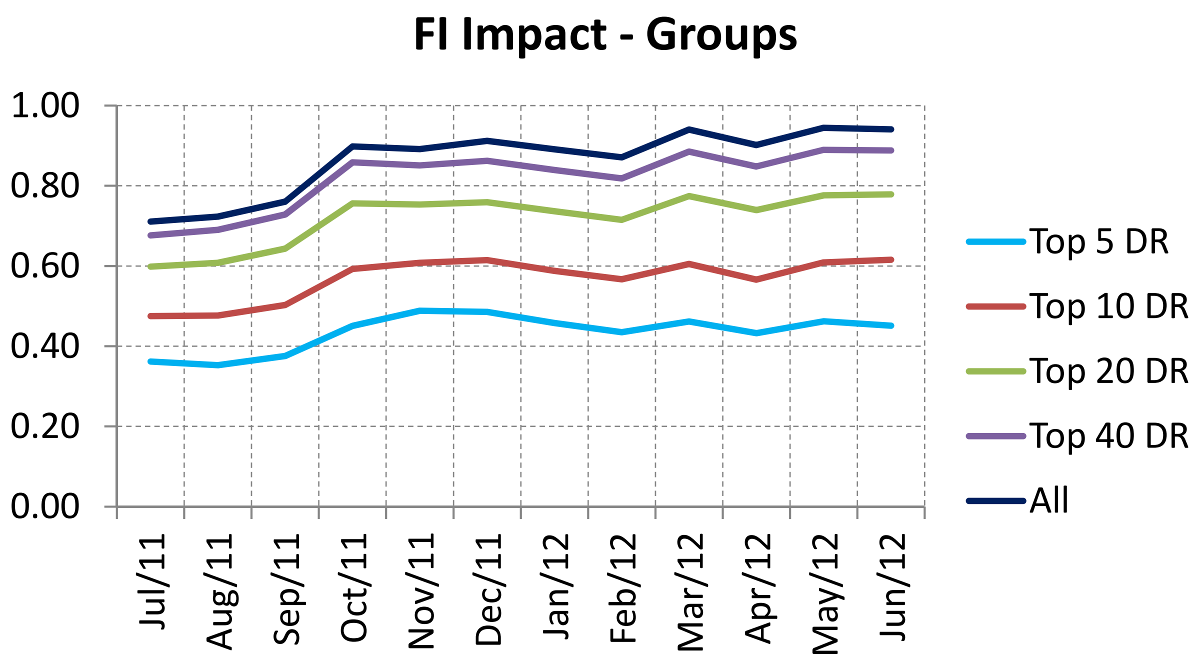

Figure 13: Impact sums of individual FIs computed for the groups of Top 5 DebtRank FIs, Top 10, Top 20, Top 40 and for all FIs.

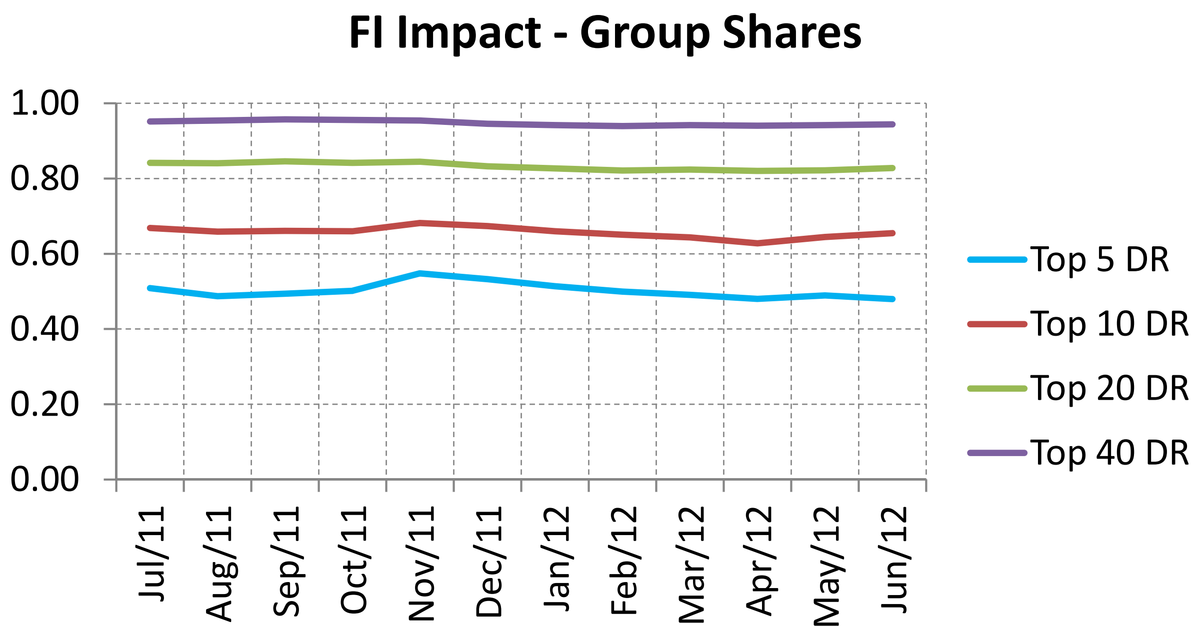

Figure 14: Share of the all FIs group sum corresponding to each group of Top DebtRank FIs.

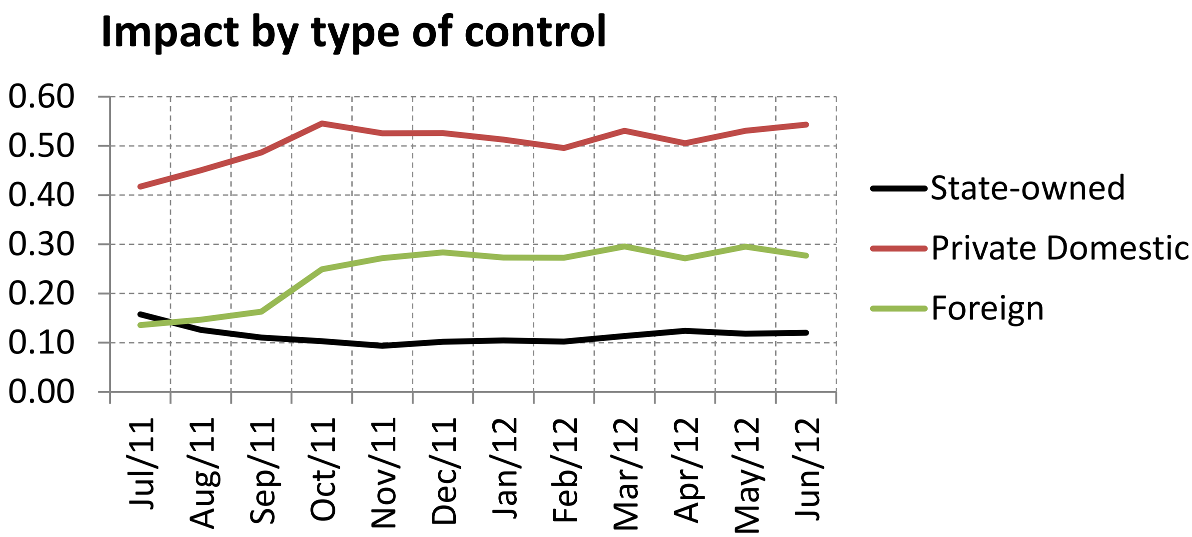

Figure 15: Impact sums of all individual FIs aggregated by type of control: State-owned, Private Domestic or Foreign.

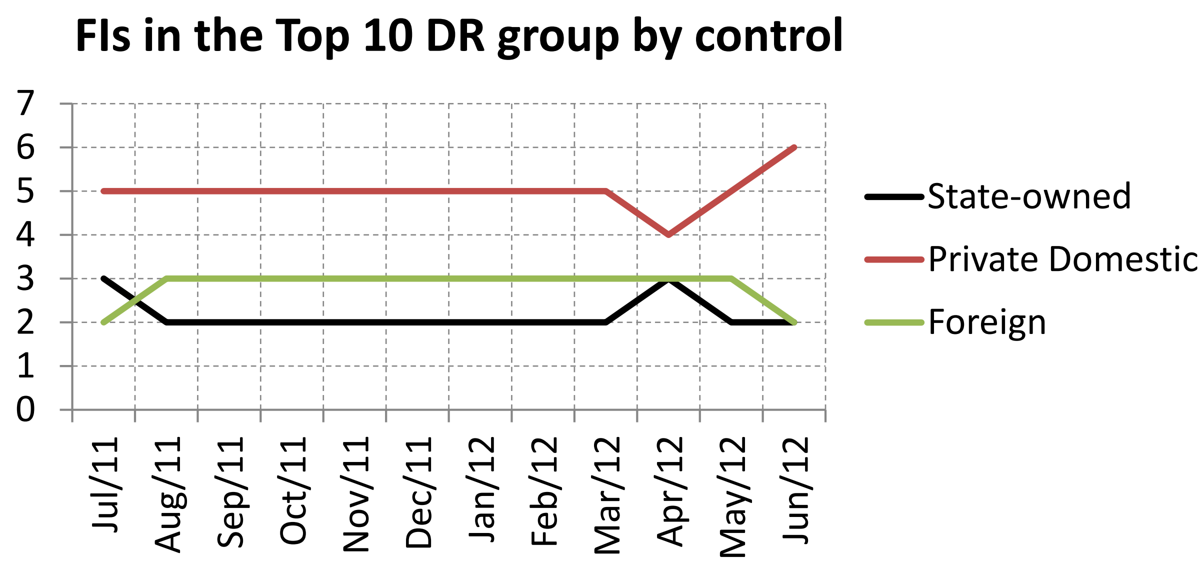

Figure 16: Number of FIs belonging to the Top 10 DebtRank group according to their type of control: State-owned, Private Domestic or Foreign.

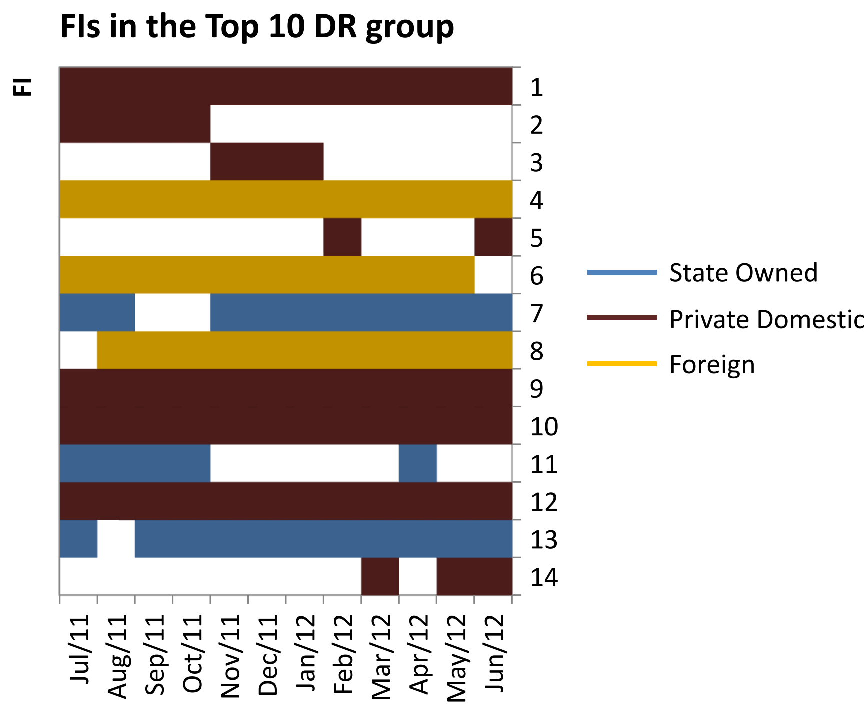

Figure 17: Type of control of the FI in each ranking of the Top 10 DebtRank group in each month.

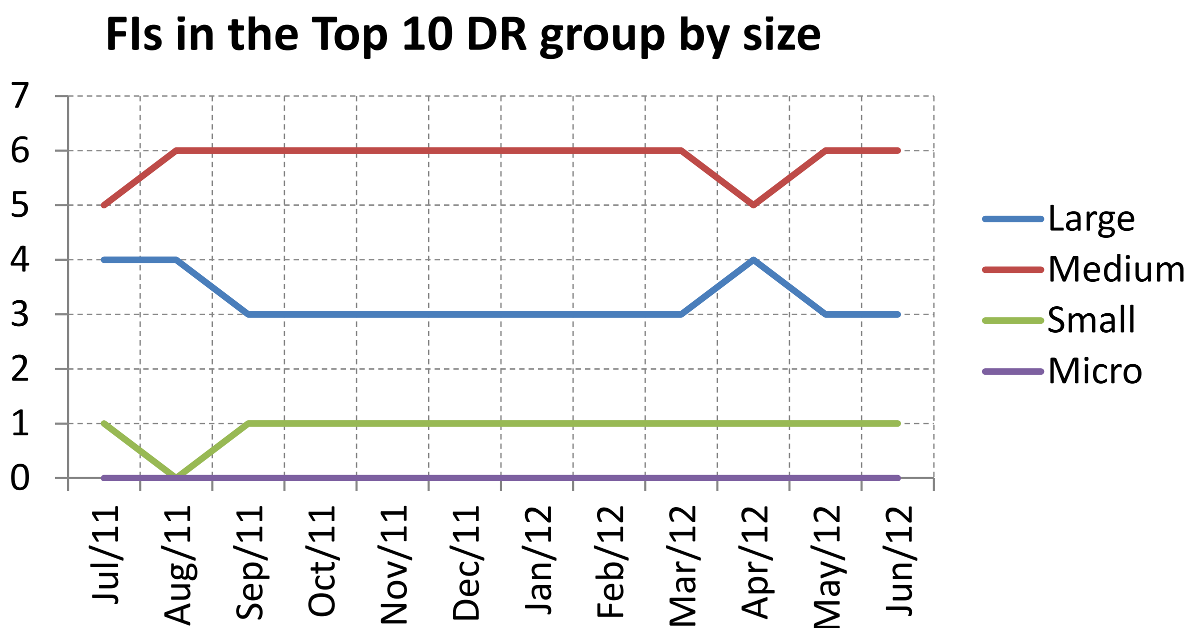

Figure 18: Number of FIs belonging to the Top 10 DebtRank group according to their size: Large, Medium, Small and Micro.

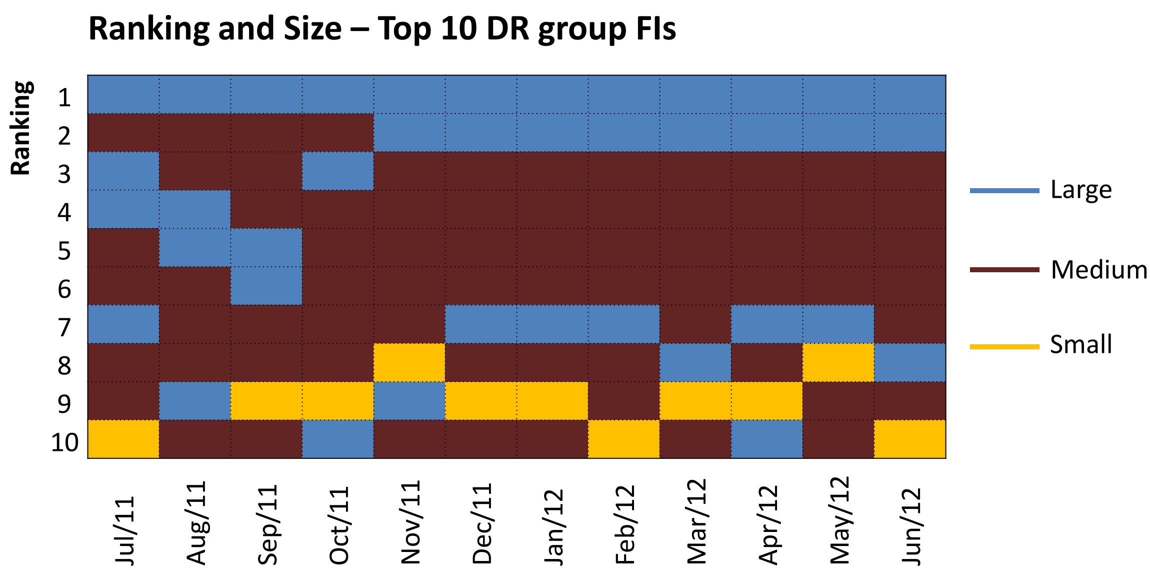

Figure 19: Size of the FI in each ranking of the Top 10 DebtRank group in each month.

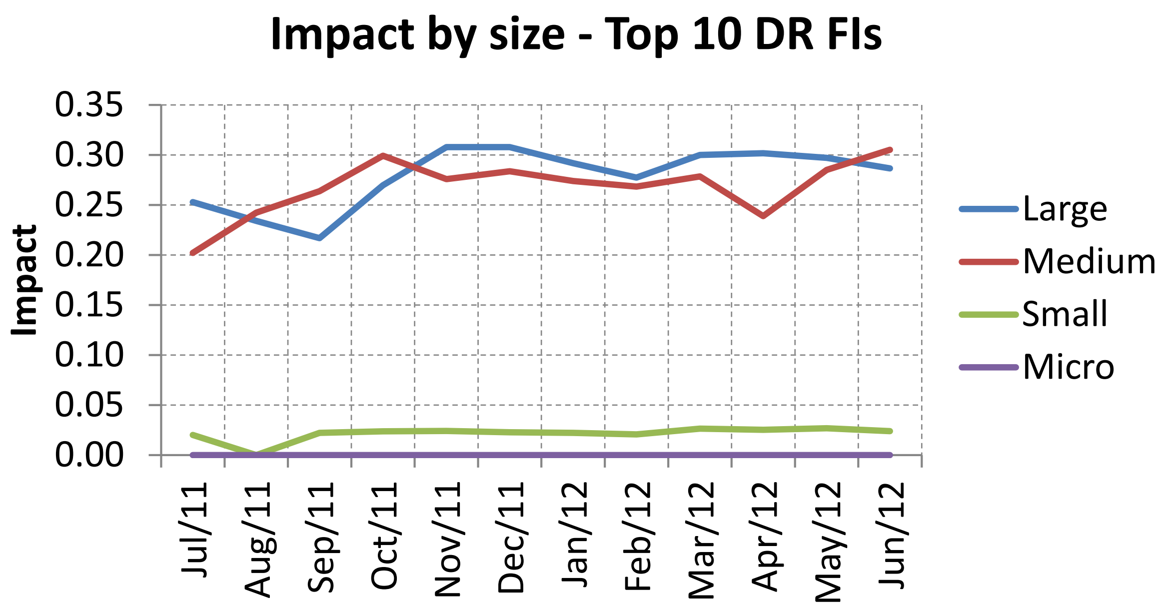

Figure 20: Impact sums of individual FIs in the Top 10 DebtRank group, aggregated by FI size: Large, Medium, Small and Micro.

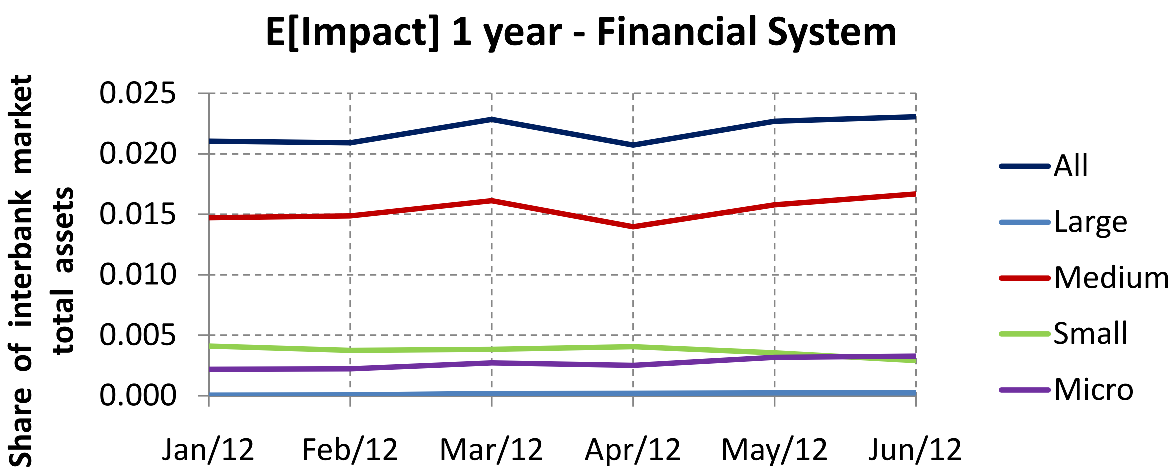

Figure 21: Expected Impact on an 1-year horizon for all the Financial System and for its subgroups of Large, Medium, Small and Micro FIs.

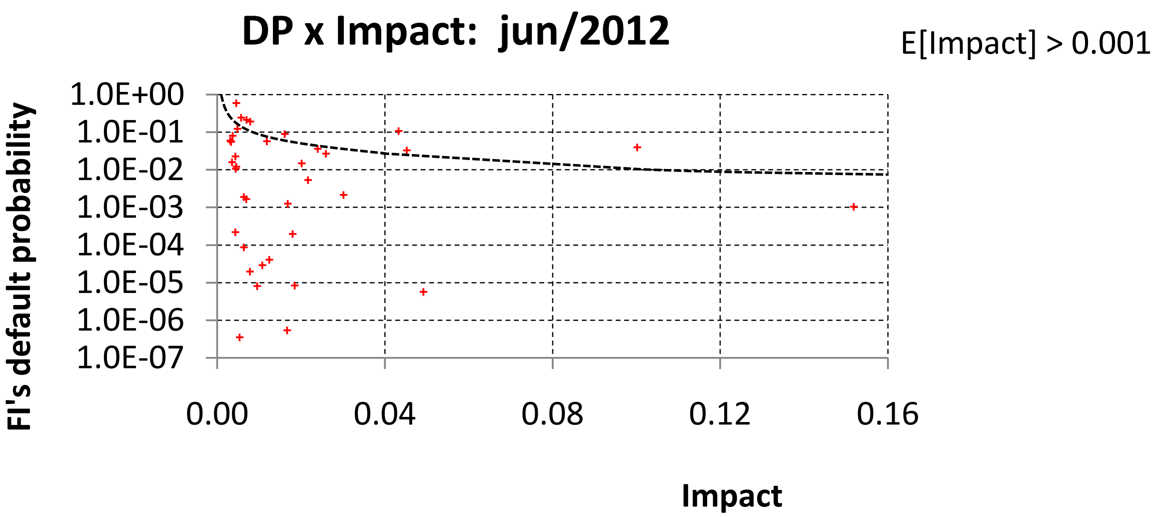

Figure 22: Default probability $\times$ impact for the Top 40 DebtRank FIs on June 2012. The FIs above the curve have expected impact above 0.001.

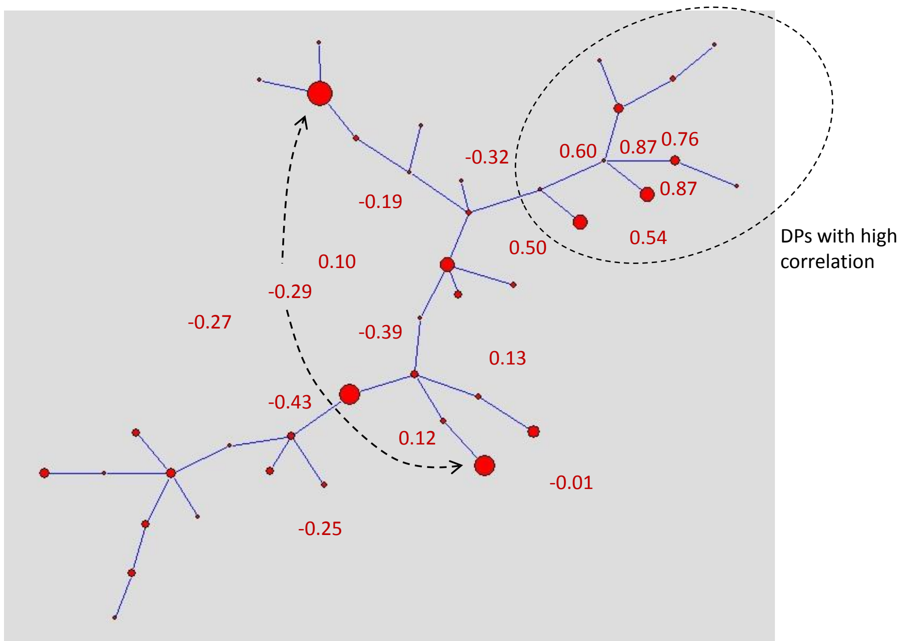

Figure 23: Minimum spanning tree with the Top 40 DebtRank FIs. The links between FIs are computed from their default probability correlation. The links are formed from the pairs of FIs with the highest default probability correlation. The FIs' size represents its DebtRank. The ellipsis in the upper right corner highlights a group of FIs which have high default probability correlation.