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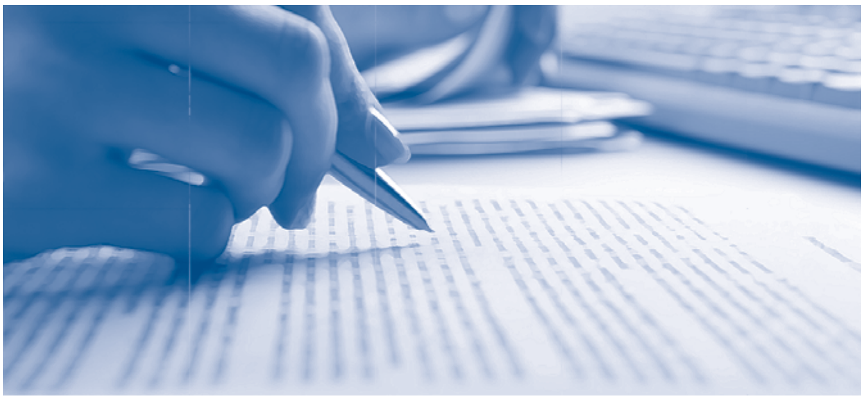

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Forecasting Multivariate Time Series under Present-Value-Model Short- and Long-run Co-movement Restrictions*

Osmani Teixeira de Carvalho Guillén† Alain Hecq‡
João Victor Issler§ Diogo Saraiva¶

Abstract

This paper has two original contributions. First, we show that PV relationships entail a weak-form SCCF restriction, as in Hecq et al. (2006) and in Athanasopoulos et al. (2011), and implies a polynomial serial correlation common feature relationship (Cubadda and Hecq, 2001). These represent short-run restrictions on the dynamic multivariate systems, something that has not been discussed before. Our second contribution relates to forecasting multivariate time series that are subject to PVM restrictions, which has a

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†Banco Central do Brasil

‡Maastricht University

§Corresponding address: João Victor Issler, Graduate School of Economics - EPGE, Getulio Vargas Foundation, Praia de Botafogo 190 s. 1100, Rio de Janeiro, RJ 22250-900, Brazil. E-mail: Joao.Issler@fgv.br. The authors gratefully acknowledge the financial suport of INCT-Brazil (CNPq and FAPERJ). We also thank Søren Johansen for bringing to our attention several of his recent papers.

¶Getulio Vargas Foundation

wide application in macroeconomics and finance. We benefit from previous work showing the benefits for forecasting when the short-run dynamics of the system is constrained. The reason why appropriate common-cycle restrictions improve forecasting is because it finds linear combinations of the first differences of the data that cannot be forecast by past information. This embeds natural exclusion restrictions preventing the estimation of useless parameters, which would otherwise contribute to the increase of forecast variance with no expected reduction in bias.

We applied the techniques discussed in this paper to data known to be subject to PV restrictions: the online series maintained and updated by Shiller at http://www.econ.yale.edu/~shiller/data.htm. We focus on three different data sets. The first includes the levels of interest rates with long and short maturities, the second includes the level of real price and dividend for the S&P composite index, and the third includes the logarithmic transformation of prices and dividends. Our exhaustive investigation of six different multivariate models reveals that better forecasts can be achieved when restrictions are applied to them. Specifically, cointegration restrictions, and cointegration and weak-form SCCF rank restrictions, as well as all the set of theoretical restrictions embedded in the PVM.

JEL: C22, C32

Keywords: forecasting, multivariate models, vector autoregression (VAR), present-value restrictions, common cycles, cointegration, interest rates, prices and dividends.

1 Introduction

Using multivariate models in economics and other sciences have been proved fruitful since they entail key inter-relationships between the variables being modelled. Unfortunately, most of these models have an abundance of free parameters, which poses a problem when they are used for forecasting, since their forecast-accuracy measures are usually outperformed by those of more parsimonious alternatives. One way to cope with this problem is to impose restrictions, reducing the number of free parameters. In economics, this is done in two different ways within a unified framework – usually, but not exclusively, a vector autoregressive (VAR) model. The first is to impose long-run relationships among the series being modelled when they trend over time, i.e., to impose cointegration restrictions; see Engle and Granger (1987). The second is to impose similarities in their short-run dynamics, i.e., to impose common-cycle restrictions either in weak or in strong form; see Engle and Kozicki (1993), Vahid and Engle (1993) and Hecq et al. (2000, 2006).

The extensive work on cointegration (e.g. Engle and Yoo, 1987 or Reinsel and Ahn, 1992, and all the literature that followed) has shown that considering and imposing long-run relationships leads to forecasting gains compared to the model in first differences (see also Clements and Hendry, 1998 or Hoffman and Rasche, 1996, *inter alia*). However, less than a handful of papers (e.g., Issler and Vahid, 2001, Vahid and Issler, 2002, Anderson and Vahid, 2011) have investigated whether additional short-run co-movement restrictions generate better forecasts. Moreover, only recently have Athanasopoulos et al. (2011) compared the relative importance of these two types of restrictions using simulations and real data, their results showing that existing short-run restrictions have a greater potential to improve forecast accuracy compared to cointegration restrictions.

Perhaps the asymmetry in the treatment of long- and short-run co-movement is due to the fact that cointegration is often interpreted in terms of economic relationships. Of course, this does not mean that short-run restrictions cannot be a consequence of economic theory. For example, Candelon and Hecq (2000) and Basistha and Startz (2008) link the former to Okun's law. Issler and Vahid (2001) give examples where the short-run behavior of consumption and income is restricted by liquidity-contraint models or by optimal-consumption models.

More importantly, short-run restrictions are implied by the present-value model studied here. So are long-run restrictions, but Campbell and Shiller (1987) have only stressed the fact that cointegration between the level of two variables (labeled Y_t and y_t in this paper) is a necessary condition for the validity of a present-value model (PV and PVM, respectively, hereafter) linking them.[1] Hence, it is often overlooked that *another* necessary condition for the PVM to hold is that the forecast error entailed by the PV model is orthogonal to the past. We refer to Hansen and Sargent (1981, 1993) and Baillie (1989) for initial work on rational expectations linked to PVMs, and Johansen and Swensen (1999, 2004, 2011), and Johansen (2000), for a recent fresh look on the subject. The basis of this result is the use of *rational expectations* in forecasting future values of variables in the PVM.

Indeed, PVMs arise from a first-order stochastic difference equation, where its

[1] Examples of Y_t and y_t are, respectively, prices and dividends for a given asset, long- and short-term interest rates, and consumption and disposable income. If they are integrated processes, they will cointegrate. See also the examples in Campbell (1987) and Campbell and Deaton (1989), *inter alia*, which are reviewed in Engsted (2002), and the interesting recent contribution of Johansen and Swensen (2011).

error term must be unforecastable regarding past information, i.e., it must have a zero conditional expectation. If this fails, the PV equation will not be valid, since it will contain an additional term capturing the (non-zero) conditional expected value of all future error terms. Cointegration imposes the transversality condition allowing to discard the limit $I(0)$ combination of Y_t and y_t. The existence of an unforecastable linear combination of the $I(0)$ series in the difference equation guarantees that the dynamic behavior of the variables in the PVM is consistent with theory. Since we need both conditions to validate PVMs, it is ideal to work with an integrated econometric framework encompassing the joint existence of these two phenomena.

This is the starting point of this article. We first show that PV relationships entail a weak-form common feature restriction, as in Hecq et al. (2006) and in Athanasopoulos et al. (2011), for the vector error-correction model (VECM) for Y_t and y_t. It also implies a polynomial serial correlation common feature relationship (Cubadda and Hecq, 2001) for the VAR representation for Δy_t and the cointegrating relationship $Y_t - \theta y_t$. These represent short-run restrictions on the dynamic system for these variables. Once we cast the PVM in these terms, it is straightforward to apply the toolkit of the *common-feature* literature for inference and testing, which has superior results vis-a-vis standard methods. This is the first original contribution of this paper.

Our second contribution relates to forecasting series that are subject to PVM restrictions, which has a wide application in macroeconomics and finance. We benefit from our previous theoretical results, especially regarding the existence of common-cyclical features in its various forms. As is well known, there has been previous work showing the forecast benefits when the short-run dynamics of the system is constrained for stationary data (Vahid and Issler, 2002), and when it is constrained for data subject to long- and short-run restrictions (Issler and Vahid, 2001, Anderson and Vahid, 2011, and Athanasopoulos et al., 2011). The reason why appropriate common-cycle restrictions improve forecasting is because they find linear combinations of the first differences of the data that cannot be forecast by past information. This embeds natural exclusion restrictions preventing the estimation of useless parameters, which would otherwise contribute to the increase of forecast variance with no expected reduction in bias. After all, forecast models should not try to forecast the unforecastable. The whole issue is obviously parsimony, but the exclusion restrictions are chosen in a way that is aligned with the final objective – to forecast the series in the system – eliminating parameters that go against that.

We show the relevance of the issues discussed above in an empirical exercise involving two sets of financial series. The first contains annual long- and short-maturity interest rates for the U.S. economy. The second contains real price and dividend for the S&P composite index and the real risk-free rate. Both data sets were extracted from the online library maintained and updated by Shiller (http://www.econ.yale.edu/~shiller/data.htm), with 142 annual observations spanning the period 1871-2012. We are careful to consider different layers of restrictions discussed in the PVM literature: long-run restrictions (cointegration), short-run restrictions (weak-form common cycles), long- and short-run restrictions jointly, and the latter with additional specific parameter restrictions implied by economic theory. Each layer corresponds to a specific restricted representation for the reduced form VAR. Forecast-accuracy measures across representations are compared to evaluate the benefits of imposing each set of restrictions. Since all restricted representations forecast the first differences of the data, but the VAR forecasts their level, we transformed VAR forecasts errors into first-difference counterparts in making final comparisons.

Our last contribution is to devise a testing strategy for PV restrictions in macroeconomics and finance incorporating more than 20 years of research on this topic. We cover several important issues. First, how to choose consistently the lag length of the VAR. Second, testing for cointegration, common cycles and weak-form common cycles. We discuss a multivariate approach based on the likelihood ratio test (canonical correlation analysis) and a single-equation heteroskedasticity robust approach (GMM). Part of our suggested strategy relies on Monte-Carlo simulation results. Finally, we also suggest integrated approaches estimating jointly the lag length of the VAR and long-run and short-run parameters as in Athanasopoulos et al. (2011), and an alternative estimating jointly long-run and short-run parameters as in Centoni, Cubadda and Hecq (2007). In order to avoid using too much space of a forecast paper with testing and estimation issues, they are discussed in the Appendix.

The rest of the paper is divided as follows. Section 2 reviews PV formulas and notations (both for the levels and the log-levels of the variables) and discusses the types of restrictions a simple present value model implies for the VECM as well as for a transformed VAR. In Section 3, we present an in-sample analysis of the data used in the forecast experiment, verifying whether or not some of the restrictions implied by economic theory hold in practice. In Section 4 we compare the forecasting gains obtained from imposing different types of PV restrictions in multivariate models. Section 5 concludes. The Appendix contains additional material on how to select the lag-length of the VAR in our context, how to implement different tests of PVMs, including their small-sample performance, and other relevant issues for examining PVM restrictions.

2 Present-value models

2.1 Basic representation in levels, long- and short-run co-movement

Consider the present value equation:[2]

$$Y_t = \theta(1-\delta)\sum_{i=0}^{\infty}\delta^i \mathbb{E}_t y_{t+i}, \tag{1}$$

which states that Y_t is a linear function of the present discounted value of expected future y_t, where $\mathbb{E}_t(\cdot)$ is the conditional expectation operator, using information up to t as the information set. In most cases Y_t and y_t are $I(1)$ variables. Examples of Y_t and y_t include, respectively: long and short-term interest rates, real stock prices and real dividends, personal consumption and disposable income, etc. (see the survey of Engsted, 2002). In this subsection, it is assumed constant expected returns with a discount factor $\delta = \frac{1}{1+r}$. The coefficient θ is a factor of proportionality. For example, $\theta = \delta/(1-\delta)$ in the price-dividend relationship; $\theta = 1$ for the interest rates case and the link with the discount factor is given by the term structure of the interest rates (see, *inter alia*, Chow, 1984; Campbell and Shiller, 1987; Johansen and Swensen, 2011). The choice of θ only impacts the value of the cointegrating vector. Hence,

[2] For simplicity, we do not include a constant term at this level of presentation as some papers do.

here, in what follows, we set its value equal to $\theta = \delta/(1-\delta)$, such that:

$$Y_t = \delta \sum_{i=0}^{\infty} \delta^i \mathbb{E}_t y_{t+i}. \tag{2}$$

Following Campbell and Shiller (1987), the actual spread is defined as:

$$S_t \equiv Y_t - \frac{\delta}{1-\delta} y_t, \tag{3}$$

where S_t is $I(0)$ if Y_t and y_t are cointegrated. Subtracting $\frac{\delta}{1-\delta} y_t$ from both sides of (2) produces the theoretical spread $S_t^{'}$:

$$S_t^{'} = \frac{\delta}{1-\delta} \sum_{i=1}^{\infty} \delta^i \mathbb{E}_t \Delta y_{t+i}. \tag{4}$$

This shows that series must be theoretically cointegrated because the right-hand side is a function of $I(0)$ terms with exponentially decreasing weights. Further, subtracting $\delta \mathbb{E}_t Y_{t+1} = \delta \sum_{i=0}^{\infty} \delta^i \mathbb{E}_t y_{t+i+1}$ from Y_t in (4), we obtain:

$$Y_t = \delta \mathbb{E}_t Y_{t+1} + \delta y_t. \tag{5}$$

From (5), if one adds and subtracts δY_t, leading to $Y_t = \delta \mathbb{E}_t \Delta Y_{t+1} + \delta Y_t + \delta y_t$ or $(1-\delta) Y_t = \delta \mathbb{E}_t \Delta Y_{t+1} + \delta y_t$, one finally obtains:

$$S_t^{''} = \frac{\delta}{1-\delta} \mathbb{E}_t \Delta Y_{t+1}. \tag{6}$$

Equation (6) gives the spread as a function of one-step ahead forecasts of ΔY_{t+1}. We can always perform the following decomposition:

$$\Delta Y_{t+1} = \mathbb{E}_t \Delta Y_{t+1} + \underbrace{(\Delta Y_{t+1} - \mathbb{E}_t \Delta Y_{t+1})}_{u_{t+1}}. \tag{7}$$

Plugging (7) into (6), and lagging the whole equation by one period we have $S_{t-1} = \frac{\delta}{1-\delta}\Delta Y_t + u_t$ or alternatively,

$$\Delta Y_t = \frac{1-\delta}{\delta} S_{t-1} + v_t \tag{8}$$

where u_t (or $v_t = -\frac{1-\delta}{\delta} u_t$) is orthogonal to the past in expectation. From (8) we also obtain:

$$\begin{aligned} (1-\delta)S_{t-1} &= \delta \Delta Y_t + (1-\delta)u_t \qquad (9) \\ S_{t-1} - \delta Y_{t-1} + \delta \frac{\delta}{1-\delta} y_{t-1} &= \delta Y_t - \delta Y_{t-1} + \left\{ \delta \frac{\delta}{1-\delta} y_t - \delta \frac{\delta}{1-\delta} y_t \right\} + (1-\delta)u_t \\ S_{t-1} &= \delta S_t + \delta \frac{\delta}{1-\delta} \Delta y_t + (1-\delta)u_t \end{aligned}$$

which gives

$$S_t = \frac{1}{\delta} S_{t-1} - \frac{\delta}{1-\delta} \Delta y_t + \varepsilon_t \tag{10}$$

with $\varepsilon_t = \frac{(1-\delta)}{\delta} u_t$.

As stressed by Campbell (1987), in the context of saving, equation (10) plays a very important role: it is the first order stochastic difference equation that generates the PVM. There are two important conditions to go from (10) to (4): cointegration delivers the transversality condition $\lim_{k\to\infty} \delta^k \mathbb{E}_t (S_{t+k}) = 0$, whereas unforecastability of ε_t regarding the past, i.e., $\mathbb{E}_t (\varepsilon_{t+j}) = 0$, for all $j > 0$, ensures that there is no additional term in the right-hand side of (4) invalidating it. The first represents a long-run restriction between Y_t and y_t. The second restricts the dynamics of the stationary representation of the system, making S_t and Δy_t specific functions of their own past alone. Thus, they can be viewed as short-run restrictions on the behavior of S_t and Δy_t. These are exactly the types of restrictions studied in the common-cycle literature. Therefore, applying the *toolkit* developed there allows a fresh view of PVMs as we show below.

Remark 1. *Johansen and Swensen (2011) discuss the properties of the three spreads* S_t*,* $S_t^{'}$*, and* $S_t^{''}$*. Their setup is slightly different than ours, since, in (1), they define the present-value relationship to be* $Y_t = \sum_{i=1}^{\infty} \delta^i \mathbb{E}_t y_{t+i}$ *instead of* $Y_t = \theta(1-\delta)\sum_{i=0}^{\infty} \delta^i \mathbb{E}_t y_{t+i}$*, i.e., they discount only future values of* y_t *and not its current value. Some authors prefer the latter to the former using the argument that, in the discrete time setup, the cash flow is accrued at the end of every period. Here, we follow Campbell and Shiller in their choice of PV formula using (1). The cointegrating vector is not affected by this choice, but the short-run dynamic-coefficient restrictions are – as we shall see in the next section. For that reason, some of our results are not identical to those in Johansen and Swensen (2011).*

2.2 Common-cyclical feature restrictions: VARs and VECMs

Assume that the bivariate system for the $I(1)$ series $(Y_t, y_t)'$ follows a VAR(p) in levels, and that $S_t = Y_t - \theta y_t$ is the stationary error-correction term. In the price-dividend case $\theta = \frac{\delta}{1-\delta}$. The corresponding vector error-correction model (VECM) representation is given by:

$$\begin{pmatrix} \Delta Y_t \\ \Delta y_t \end{pmatrix} = \Gamma_1 \begin{pmatrix} \Delta Y_{t-1} \\ \Delta y_{t-1} \end{pmatrix} + ... + \Gamma_{p-1} \begin{pmatrix} \Delta Y_{t-p+1} \\ \Delta y_{t-p+1} \end{pmatrix} + \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} S_{t-1} + \begin{pmatrix} \eta_{1t} \\ \eta_{2t} \end{pmatrix}, \tag{11}$$

where we assume that the disturbance terms are white noise and that conditions for avoiding $I(2)$-ness are met. The $\Gamma_i s$ are the short-run coefficient matrices, and α_1 and α_2 are the loadings on the error-correcting term.

As is well known, PV relationships imply restrictions on dynamic models of the data. Campbell and Shiller (1987) and others have exploited the fact that VARs have cross-equation restrictions. Here, however, we exploit a different nature of these restrictions – the fact that there are also reduced-rank restrictions for the VECM (11).

Proposition 2. *If the elements of* $(Y_t, y_t)'$ *obey a PV relationship as in (9), i.e.,* $S_{t-1} = \frac{\delta}{1-\delta}\Delta Y_t + u_t$, *then, their VECM obeys a weak-form common feature relationship (see Hecq et al., 2006, and Athanasopoulos et al., 2011): there exists a* 1×2 *vector* γ' *such that* $\gamma'\Gamma_1 = \gamma'\Gamma_2 = ... = \gamma'\Gamma_{p-1} = 0$, *but* $\gamma' \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \neq 0$. *Moreover,* $\gamma' = (1 : 0)$, *the first row of every* Γ_i, $i = 1...p-1$, *must be zero, and the following restriction must also be met:* $\alpha_1 = \frac{1-\delta}{\delta}$.

The usual cross-equation restriction within the VAR and proposed by Campbell and Shiller (1987) can also be seen from a transformed VAR on S_t and Δy_t; see Johansen and Swensen (2011). To go from the VECM (11) to the transformed VAR representation we use $C = \begin{bmatrix} 1 & -\theta \\ 0 & 1 \end{bmatrix}$, the 2×2 nonsingular matrix formed by stacking the transpose of the cointegrating vector $\begin{bmatrix} 1 & -\theta \end{bmatrix}$ and the selection vector $\begin{bmatrix} 0 & 1 \end{bmatrix}$, such that $C \begin{pmatrix} \Delta Y_t \\ \Delta y_t \end{pmatrix} = \begin{pmatrix} \Delta S_t \\ \Delta y_t \end{pmatrix}$. Premultiplying both sides of (11) by C, and solving for S_t and Δy_t, we obtain:

$$\begin{pmatrix} S_t \\ \Delta y_t \end{pmatrix} = \begin{pmatrix} \underline{\Gamma_{11}}(L) & \underline{\Gamma_{12}}(L) \\ \underline{\Gamma_{21}}(L) & \underline{\Gamma_{22}}(L) \end{pmatrix} \begin{pmatrix} S_{t-1} \\ \Delta y_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix} \tag{12}$$

where $\underline{\Gamma_{11}}(L)$ and $\underline{\Gamma_{21}}(L)$ are polynomials of order $p-1$ and $\underline{\Gamma_{12}}(L)$ and $\underline{\Gamma_{22}}(L)$ are polynomials of order $p-2$. Indeed, one important issue is to note is that the transformed VAR (12) is a VAR of order p both in S_t and in Δy_t in which the two coefficients of Δy_{t-p} are zero. Cross-equation restrictions for the system are imposed on the coefficient matrices of $\underline{\Gamma}(L) = \begin{pmatrix} \underline{\Gamma_{11}}(L) & \underline{\Gamma_{12}}(L) \\ \underline{\Gamma_{21}}(L) & \underline{\Gamma_{22}}(L) \end{pmatrix} = \underline{\Gamma}_1 + \underline{\Gamma}_2 L + ... + \underline{\Gamma}_p L^{p-1}$ in (12).

We have the following proposition.

Proposition 3. *A PVM as in (10), i.e.,* $S_t = \frac{1}{\delta} S_{t-1} - \frac{\delta}{1-\delta} \Delta y_t + \varepsilon_t$, *implies a polynomial serial-correlation common feature relationship (see Cubadda and Hecq, 2001) for the transformed VAR (12): there exists a vector* $\tilde{\gamma}_0'$ *such that* $\tilde{\gamma}_0' \underline{\Gamma}_2 = ... = \tilde{\gamma}_0' \underline{\Gamma}_p = 0$, *with* $\tilde{\gamma}_0' \underline{\Gamma}_1 = \tilde{\gamma}_1' \neq 0$. *Moreover in the PVM* $\tilde{\gamma}_0' = (1 : \frac{\delta}{1-\delta})$ *and* $\tilde{\gamma}_1' = (-\frac{1}{\delta} : 0)$.

Thus, a PVM entails cointegration and additional orthogonality conditions associated with reduced rank restrictions in VECMs or transformed VARs. One of the possible explanations for observing a rejection of the PVMs is the use of cross-equation restrictions that impose both reduced-rank restrictions and particular values on the parameters. Misspecifications such as proxy variables or measurement errors can affect the value of the parameters, leaving unaffected the reduced-rank restrictions. As an example, instead of the PV representation $Y_t = \theta(1 - \delta) \sum_{i=0}^{\infty} \delta^i \mathbb{E}_t y_{t+i}$ one can find in the literature that the series Y_t is a function of the future discounted expected value of y_t such that $Y_t = \sum_{i=1}^{\infty} \delta^i \mathbb{E}_t y_{t+i}$.[3] This slight change is not innocuous as we show next. To see that, apply the algebra used before

[3] Johansen and Swensen (2011) as well as Campbell, Lo and Mackinlay (1996) use that formulation when they consider the stock price at the end of the period.

to $Y_t = \sum_{i=1}^{\infty} \delta^i \mathbb{E}_t y_{t+i}$ to obtain the following expressions:

$$\begin{aligned} \Delta Y_t &= -\Delta y_t + \frac{1-\delta}{\delta} S_{t-1} + u_t, &(13)\\ \text{where } \gamma' &= (1:1) \text{ and } \alpha_1 = \frac{1-\delta}{\delta} \text{ in Proposition 1.} &(14) \end{aligned}$$

$$\begin{aligned} S_t &= -\frac{1}{(1-\delta)} \Delta y_t + \frac{1}{\delta} S_{t-1} + v_t, &(15)\\ \text{where } \tilde{\gamma}_0' &= (1 : \frac{1}{1-\delta}) \text{ and } \tilde{\gamma}_1' = (-\frac{1}{\delta} : 0) \text{ in Proposition 2.} &(16) \end{aligned}$$

What emerges now is that the unpredictable linear combinations involve three variables: ΔY_t, Δy_t, and S_t, both in the VECM and the transformed VAR. Moreover the values of the parameters are now different from before – the weights used in the linear combinations (13) and (15) differ from the ones in (8) and (10), respectively.

Put differently, regarding the use of $Y_t = \sum_{i=1}^{\infty} \delta^i \mathbb{E}_t y_{t+i}$ versus $Y_t = \theta(1-\delta) \sum_{i=0}^{\infty} \delta^i \mathbb{E}_t y_{t+i}$, respectively, yields the following orthogonality conditions for each specific difference equation:

$$\begin{aligned} \mathbb{E}_{t-1}\left[\Delta Y_t + \Delta y_t - \frac{1-\delta}{\delta} S_{t-1}\right] &= 0, \text{ vs. } \mathbb{E}_{t-1}\left[\Delta Y_t - \frac{1-\delta}{\delta} S_{t-1}\right] = 0,\\ \mathbb{E}_{t-1}\left[S_t + \frac{1}{(1-\delta)} \Delta y_t - \frac{1}{\delta} S_{t-1}\right] &= 0, \text{ vs. } \mathbb{E}_{t-1}\left[S_t + \frac{\delta}{1-\delta} \Delta y_t - \frac{1}{\delta} S_{t-1}\right] = 0. \end{aligned}$$

Despite the differences in parameter values in the linear combinations above, the existence of a reduced-rank model is not affected by how one writes the PV equation linking Y_t and y_t. Hence, the reduced-rank properties of the VECM and of the transformed VAR are invariant to this choice: in both cases, there exists weak-form common features for the VECM and the PSCCF for the transformed VAR.

2.3 Constant versus variable expected returns: levels versus logs

Campbell and Shiller's (1987) model for the level of prices (Y_t) and dividends (y_t) is consistent with a very restrictive assumption – that the expected return of a given stock is constant through time:

$$\mathbb{E}_t\left[R_{t+1}\right] = R, \tag{17}$$

where

$$R_{t+1} \equiv \frac{P_{t+1} + D_{t+1}}{P_t} - 1, \tag{18}$$

where P_t and D_t denote, respectively, the price and the dividend of a given stock.

In two subsequent papers, Campbell and Shiller (1988a,b) developed an alternative representation for prices and dividends, in which (17) needs not hold, so being consistent with the idea of time-varying returns. This alternative representation uses the logarithm of prices and dividends:

$$h_{t+1} \equiv \log(1 + R_{t+1}) = \log(P_{t+1} + D_{t+1}) - \log(P_t), \tag{19}$$

arriving at:

$$h_{t+1} = p_{t+1} - p_t + \log(1 + \exp(d_{t+1} - p_{t+1})), \tag{20}$$

where lower-case variables represent the respective logarithmic transformation of the original variable.

They use a first-order taylor expansion in (20), to get:

$$
\begin{aligned}
h_{t+1} &\approx k + \rho p_{t+1} + (1-\rho) d_{t+1} - p_t && (21) \\
&= k + \rho p_{t+1} + (1-\rho) d_{t+1} - p_t + d_t - d_t && (22) \\
&= k + \rho p_{t+1} + d_{t+1} - d_t - \rho d_{t+1} + d_t - p_t && (23) \\
&= k + \rho p_{t+1} - \rho d_{t+1} + \Delta d_{t+1} + d_t - p_t && (24) \\
&= k + \rho \left(p_{t+1} - d_{t+1} \right) + \Delta d_{t+1} + \left(d_t - p_t \right) && (25)
\end{aligned}
$$

where $\rho \equiv \frac{1}{(1+\exp(\overline{d-p}))}$, and $\overline{d-p}$ is the average across time of $d_{t+1} - p_{t+1}$, and $k \equiv -\log(\rho) - (1-\rho)\log(1/\rho - 1)$.

Notice that we can solve (25) for $(d_t - p_t)$, yielding an exact stochastic first-order difference equation for it,

$$
(d_t - p_t) = -k + h_{t+1} - \Delta d_{t+1} - \rho \left(p_{t+1} - d_{t+1} \right) + \varepsilon_{t+1}, \qquad (26)
$$

where ε_{t+1} is an approximation error. Under the assumption that $\mathbb{E}_t \left(\varepsilon_{t+j} \right) = 0$, for all $j > 0$, equation (26) can be solved forward to yield a logarithmic version of equation (4):

$$
d_t - p_t = -\frac{k}{1-\rho} + \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \left[h_{t+1+j} - \Delta d_{t+1+j} \right]. \qquad (27)
$$

Campbell and Shiller (1988a) argue that "there is no economic content in equation (27)". To get economic content they impose a restriction on the behavior and dynamics of h_t:

$$
\mathbb{E}_t h_{t+1} = \mathbb{E}_t r_{t+1} + c, \qquad (28)
$$

i.e., that the excess-return of a stock, vis-a-vis the real risk-free rate r_t, is constant.[4]

[4]Equation (28) implies the existence of a common cycle for h_t and r_t.

If r_t is observable, (27) and (28) yield a testable econometric model:

$$d_t - p_t = \frac{c-k}{1-\rho} + \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \left[r_{t+1+j} - \Delta d_{t+1+j} \right]. \tag{29}$$

2.4 Common-cyclical feature restrictions: the logarithmic version

To test the log-linear present value model embedded in (29), we use the tridimensional system for $X_t = (p_t, d_t, r_t)'$. Notice first that (28) implies that r_t is $I(0)$, given that $h_t =$ is $I(0)$. This yields the first cointegrating vector for the system in X_t. Given that r_t is $I(0)$, from (29) $d_t - p_t$ is $I(0)$ as well, yielding the second cointegrating vector in the system.

The VECM$(p-1)$ reads as:

$$\begin{bmatrix} \Delta p_t \\ \Delta d_t \\ \Delta r_t \end{bmatrix} = \begin{bmatrix} \gamma_1 & \gamma_4 \\ \gamma_2 & \gamma_5 \\ \gamma_3 & \gamma_6 \end{bmatrix} \begin{bmatrix} (d_{t-1} - p_{t-1}) \\ r_{t-1} \end{bmatrix} + \begin{pmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 \end{pmatrix} \Delta X_{t-1} +$$
$$\cdots + \begin{pmatrix} a_{11}^{p-1} & a_{12}^{p-1} & a_{13}^{p-1} \\ a_{21}^{p-1} & a_{22}^{p-1} & a_{23}^{p-1} \\ a_{31}^{p-1} & a_{32}^{p-1} & a_{33}^{p-1} \end{pmatrix} \Delta X_{t-p-1} + \epsilon_t \tag{30}$$

Disregarding an irrelevant constant term, the PVM implies that $\mathbb{E}_{t-1}[h_t - r_t] = 0$. From equation (25), we can approximate h_t as $h_t = \rho \Delta p_t + (1-\rho)(d_{t-1} - p_{t-1})$. Consequently in the VECM, if we pre-multiply the system (30) by $\begin{pmatrix} \rho & 0 & -1 \end{pmatrix}$, in order to obtain an expression for $h_t - r_t$, we must have:

$$\underbrace{\begin{pmatrix} \rho & 0 & -1 \end{pmatrix} \Delta X_t}_{\rho \Delta p_t - \Delta r_t} = -(1-\rho)(d_{t-1} - p_{t-1}) + r_{t-1} + \begin{pmatrix} \rho & 0 & -1 \end{pmatrix} \epsilon_t \tag{31}$$

which is equivalent to

$$\underbrace{\rho\Delta p_t + (1-\rho)(d_{t-1} - p_{t-1})}_{h_t} - r_t = \begin{pmatrix} \rho & 0 & -1 \end{pmatrix} \epsilon_t, \tag{32}$$

which implies $\mathbb{E}_{t-1}[h_t - r_t] = 0$. These theoretical conditions restrict the VECM parameters as follows:

$$\begin{pmatrix} \rho & 0 & -1 \end{pmatrix} \begin{pmatrix} a_{11}^i & a_{12}^i & a_{13}^i \\ a_{21}^i & a_{22}^i & a_{23}^i \\ a_{31}^i & a_{32}^i & a_{33}^i \end{pmatrix} = 0, \; i = 1, ..., p-1, \text{ and,} \tag{33}$$

$$\begin{pmatrix} \rho & 0 & -1 \end{pmatrix} \begin{bmatrix} \gamma_1 & \gamma_4 \\ \gamma_2 & \gamma_5 \\ \gamma_3 & \gamma_6 \end{bmatrix} \begin{bmatrix} (d_{t-1} - p_{t-1}) \\ r_{t-1} \end{bmatrix} = \underbrace{(\rho\gamma_1 - \gamma_3)}_{-(1-\rho)}(d_{t-1} - p_{t-1}) + \underbrace{(\rho\gamma_4 - \gamma_6)}_{1} r_{t-1}. \tag{34}$$

The last set of restrictions in (34) are:

$$\gamma_1 = \frac{\gamma_3 - (1-\rho)}{\rho}, \text{ and,} \tag{35}$$

$$\gamma_4 = \frac{\gamma_6 + 1}{\rho}. \tag{36}$$

Given the estimates of γ_3, γ_6, and $\rho \equiv \frac{1}{(1+\exp(\overline{d-p}))}$, we can obtain the corresponding values of γ_1 and γ_4, consistent with (29). We now summarize these results with the following proposition.

Proposition 4. *If the elements in $X_t = (p_t, d_t, r_t)'$ obey a PVM as in (29) and (28), the latter leading to $h_t = r_t + \varepsilon_t$, where $h_t = \rho\Delta p_t + (1-\rho)(d_{t-1} - p_{t-1})$, and $\mathbb{E}_{t-1}[\varepsilon_t] = 0$, then, their VECM obeys a weak-form common feature relationship (Hecq et al., 2006, and Athanasopoulos et al., 2011): there exists a 1×3 vector*

$\gamma' = \begin{pmatrix} \rho & 0 & -1 \end{pmatrix}$, such that $\gamma' \begin{pmatrix} a_{11}^i & a_{12}^i & a_{13}^i \\ a_{21}^i & a_{22}^i & a_{23}^i \\ a_{31}^i & a_{32}^i & a_{33}^i \end{pmatrix} = 0$, $i = 1, ..., p-1$, implying that the first row of $\begin{pmatrix} a_{11}^i & a_{12}^i & a_{13}^i \\ a_{21}^i & a_{22}^i & a_{23}^i \\ a_{31}^i & a_{32}^i & a_{33}^i \end{pmatrix}$ is proportional to the last row $(1/\rho)$; that $\gamma' \begin{bmatrix} \gamma_1 & \gamma_4 \\ \gamma_2 & \gamma_5 \\ \gamma_3 & \gamma_6 \end{bmatrix} \neq 0$, with first-row elements being restricted as follows: $\gamma_1 = \frac{\gamma_3 - (1-\rho)}{\rho}$, and $\gamma_4 = \frac{\gamma_6 + 1}{\rho}$.

Finally, as was the case for the same series in levels, i.e., P_t and D_t, slight changes on the assumptions of when dividends are accrued (beginning, middle or end of period t) influence the short-run parameter restrictions imposed on the multivariate system for $X_t = (p_t, d_t, r_t)'$. This is not a trivial issue, especially because we are not dealing with firm data for prices and dividends. On the contrary, we are dealing with an aggregate of several firms, each one having its own dividend policy – some of which are varying across time, making it very difficult to know exactly what are the appropriate short-run parameter restrictions that should be imposed in PVMs. Although short-run parameter restrictions are not invariant to changes in dividend policy, as discussed at the end of Section 2.2, rank restrictions discussed in Propositions 1 and 3 are invariant to them. We take this explicitly into account when devising the forecast models considered in the forecast experiment.

3 In-sample analysis of the data used in the forecast experiment

Here, we analyze three different data sets which are later used in the forecast experiment. The first includes the levels of interest rates with long and short maturities, labelled i_{lr} and i_{sr}, respectively. The second includes the level of real price and dividend for the S&P composite index, labelled P_t and D_t, and the last involves the logarithmic transformation of prices and dividends, $p_t = \ln P_t$ and $d_t = \ln D_t$, with the inclusion of the real risk-free rate, labelled r_t. Data is collected for the period 1871-2012 by Shiller. Results are presented in Table 1, whereas the econometric tools used in this section are discussed at great length in the Appendix.

For each data set, we use the Hannan-Quinn information criteria to determine the lag length of the VAR in levels. This is reported in the third column of Table 1. Columns (4), (5) and (6) refers to as the cointegration analysis. We do not reject the null of no long-run relationships when the levels are used but the sensible value of the discount factor may lead to the conclusion that we have a power issue. There is a clear indication of cointegration for the interest rates as well as for prices/dividends in logs. Notice that we should have found two cointegrating vectors for the system with $(p_t, d_t, r_t)'$. Strictly speaking, we only found one cointegrating vector at 5% significance, albeit the test statistics is very to close to the critical value at that level. So, imposing two vectors could be a possibility. As far as common-cyclical features are concerned, results in columns (7) to (9) show a clear indication of weak-form common features for interest rates; we also conclude that there is a common feature vector for the system $(P_t, D_t)'$ if we use the robust GMM test J_2. For the

Table 1: Long and short-run properties of the series in the forecast experiment

			Cointegration			Common Cycle		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
System	Ends	VAR(p)	H_0	cv$_{5\%}$	$\hat{\theta}$	ξ_{LR}	J_2	cv$_{5\%}$
(P_t, D_t)	2012	3	$r = 0$: 10.47	15.49	60.75	9.76	5.65	7.81
	2007	2	$r = 0$: 12.67	15.49	69.76	10.46	1.69	3.84
(p_t, d_t, r_t)	2012	1	$r = 0 : 78.74$ $r = 1 : 14.58$	29.79 15.49	1.764	$\exists$ SCCF	$\exists$ SCCF	–
	2007	1	$r = 0 : 75.18$ $r = 1 : 14.37$	29.79 15.49	1.802	$\exists$ SCCF	$\exists$ SCCF	–
(i_{lr}, i_{sr})	2012	2	$r = 0$: 31.52	15.49	1.071	0.12	0.10	3.84
	2007	2	$r = 0$: 31.66	15.49	1.096	0.02	0.02	3.84

system with $(p_t, d_t, r_t)'$, our optimal choice of model is the VECM(0), for which there always exist SCCFs.

Therefore, we were able to find empirically that data subject to the theory of PVMs conform to some of the restrictions implied by theory. In our forecast experiment below, we ask a different question: whether or not imposing these restrictions in unrestricted multivariate models for that same data leads to an improvement of standard forecast-accuracy measures. This experiment have not only bearings on theory, but on practical issues as well.

4 Out-of-sample forecasting

4.1 Forecasting strategies imposing different co-movement restrictions

We describe next the different forecast strategies used in this paper, each of them imposing a different set of co-movement restrictions on the unrestricted VAR, our benchmark forecasting model:

1. **VAR(p) in levels (benchmark):** Select p using the Hannan-Quinn (HQ)

criterion, estimate the system by conditional maximum likelihood (ML) and use the results to forecast the variables in the system up to h periods ahead.

2. **VECM(HQ-PIC):** This is the VECM possibly restricted (but not necessarily) by cointegration and/or by weak-form serial-correlation common features: select jointly p, the rank of the short-run matrices, and the cointegrating rank, by the combination of the use of the posterior information criterion (PIC) and the HQ criterion as suggested by Athanasopoulos et al. (2011), further estimating all parameters by their two-step conditional ML. Forecast the variables in the system up to h periods ahead.

3. **VECM(HQ-J):** This is the VECM using solely the PVM-cointegration restriction. Select p by the HQ criterion, impose the cointegrating-rank restriction consistent with the PVM. Conditional on that restriction, estimate the cointegrating vector consistently (Johansen, 1991), further performing conditional ML estimation using estimated cointegrating vectors. Forecast the variables in the system up to h periods ahead.

4. **VECM(HQ)Rank:** This is the VECM using solely the PVM cointegration and weak-form serial-correlation-common-feature rank restrictions. Select p by the HQ criterion, where we impose the rank restrictions for cointegration and weak-form serial-correlation-common-feature consistent with the PVM. Estimate all parameters by conditional ML and forecast the variables in the system up to h periods ahead.

5. **PV model:** This is the VECM using the PVM cointegration and weak-form serial-correlation-common-feature rank restrictions, in addition to the theo-

retical restrictions discussed in Proposition 2. Select p by the HQ criterion, estimate all parameters by conditional ML, imposing the restrictions outlined in Proposition 1, where we use the cointegrating vector estimate of θ, $\widehat{\theta}$ (T-consistent), to constrain α_1, as $\widehat{\alpha}_1 = \frac{1-\widehat{\delta}}{\widehat{\delta}} = 1/\widehat{\theta}$. From the quasi-structural form we recover the reduced-form and forecast the variables in the system up to h periods ahead.

6. **Log PV model:** This is the VECM for $X_t = (p_t, d_t, r_t)'$, using the PVM cointegration and weak-form serial-correlation-common-feature rank restrictions, in addition to the theoretical restrictions discussed in Proposition 4. Select p by the HQ criterion. Estimate the last two reduced-form equations under the previous restrictions and set $\widehat{\rho} \equiv \frac{1}{(1+\exp(\overline{d-p}))}$, where $\overline{d-p}$ is the time-average of $d_t - p_t$. With the last two equations of reduced-form estimates and $\widehat{\rho}$, we assemble the quasi-structural form, recover the reduced-form, and then forecast the variables in the trivariate system for $X_t = (p_t, d_t, r_t)'$ up to h periods ahead. Loss functions here are computed vis-a-vis the logged variables, i.e., vis-a-vis the variables in $X_t = (p_t, d_t, r_t)'$.

In the list above, we compare models with different layers of restrictions, starting with the unrestricted model in (1), all the way through the PVM-restricted models in (5) and (6), respectively in levels and in log-levels. There is a rationale for choosing each of these different models. Model (2) is the preferred choice of Athanasopoulos et al. (2011), when the series being modelled are subject to long- and short-run restrictions. This choice is data driven, since we let information criteria (PIC and HQ) choose the cointegranting and the short-run-matrix rank, which is not imposed *a priori.* Models (3) and (4) impose (i.e. without testing for the existence of those restrictions) respectively cointegration and cointegration and weak-form serial-correlation-common-feature rank restrictions. These restrictions are motivated by the theory of PV models for trending data. Notice, however, that model (4) refrains from imposing the parameter restrictions listed in Proposition 1, but only imposes the rank restrictions. This is due to the fact that parameter restrictions are sensitive to small changes in the assumption underlying PV models. As discussed above, one of these assumptions is related to the timing in which dividends are accrued in the price-dividend model; see Remark 1 and the discussion at the end of Section 2.2. Finally, models (5) and (6) are completely theory based and impose all restrictions listed either in Proposition 1 or in Proposition 3. Comparisons of models (5) and (6) with the unrestricted VAR and with model (2) can answer whether imposing *structural* restrictions helps in forecasting, settling – at least from the point-of-view of forecasting using PVM restrictions – a dispute between theory-based econometrics (structural form) and *atheoretical* (reduced form) econometric models.[5]

For the empirical analyses we use the online series maintained and updated by Shiller at http://www.econ.yale.edu/~shiller/data.htm. The estimation details for the forecasting models are as follows. First, we divide our total sample in "estimation sample" and "forecasting sample." Since the *great recession* (2008-2009) has had a huge influence on asset prices and on the prices of bonds (interest rates), we considered two separate forecast samples: the first ending in 2007, just prior to the great recession, and the second ending in 2012, with all available information up to now. We set $h = 1, 2, ..., 12$ years in the forecasting exercise. This enables

[5] About the dispute between reduced- and structural-form in econometrics, see the recent paper by Keane (2010) and further comments on it.

measurement of short- and medium-term forecast accuracy (h between 1 and 5 years) as well as long term ($h > 10$ years). When forecasting until 2007, we have 70 observations for the estimation sample (from 1871-1940) and 67 observations for the forecasting sample (1941-2007).[6] When forecasting until 2012, the estimation sample has 75 observations (from 1871-1945) and 67 observations for the forecasting sample (1946-2012). Estimation is performed with a rolling window, kept constant throughout the out-of-sample exercise.

The forecast accuracy of all restricted models are compared to that of the VAR in levels. We use the ratio of the root-mean-squared-forecast error for each model (or variable in them) vis-a-vis that of the VAR in levels – our benchmark:

$$RRMSFE_h^M = \frac{RMSFE_h^M}{RMSFE_h^{VAR}}, \tag{37}$$

where $RRMSFE_h^M$ is the root-mean-squared-forecast error (RMSFE) statistic of model M, relative to that of the unrestricted VAR, for h step-ahead forecasting. All comparisons are made using the embedded first-difference forecast errors.

We want to be able to distinguish the forecast accuracy of models (1)-(6), asking whether their accuracy measures are statistically equal or not. We do this using the unconditional predictive ability test of Giacomini and White (2006), comparing each model M with the unrestricted VAR, and comparisons across all other models as well – not shown to save space, but available upon request.

[6] Notice that, the number of out-of-sample observations differs for $h = 1$, $h = 2$, all the way to $h = 12$: for the latter it is 56, while it is 67 for the former.

4.2 Forecasting results

We have forecast results for three different data sets. The first is regarding the levels of interest rates with long and short maturities, labelled i_{lr} and i_{sr}, respectively, where theory assumes the long rate to be the expected PV of the discounted short rate. The second is regarding the level of real price and dividend for the S&P composite index, labelled P_t and D_t, where price should be the expected PV of the discounted dividend stream. The last involves the logarithmic transformation of prices and dividends, $p_t = \ln P_t$ and $d_t = \ln D_t$, respectively, which PV analysis requires the inclusion of the real risk-free rate, labelled r_t.

We computed the relative measure of forecast accuracy (RMSFE) of model M vis-a-vis the VAR, described in (37). Since the all restricted representations (models (2)-(6)) forecast the first differences of the data, but the VAR (model (1)) forecasts their level, we transform the VAR forecasts errors into first-difference errors in order to compute the ratio in (37). Following the empirical financial literature, which relies much more on individual-data results, we focus on forecast measures for the individual variables instead of those for the system as a whole – a good example being Patton, Ramadorai, and Streatfield (2013).

In Tables 2 through 5, we present forecast results for i_{sr} and i_{lr}. When we exclude the great recession period – forecasts up to 2007, for the short rate, the PV model (5) dominates almost at all horizons, although in the short- and medium-horizon, strategy (4, VECM(HQ)Rank) dominates. For the long rate, strategy (4, VECM(HQ)Rank) dominates at all horizons, although strategy (3, VECM(HQ-J)) and PV model (5) perform well on occasion. If we extend the forecast period to 2012

Table 2: Relative RMSFE of restricted models vs VAR for i_{sr}. Forecast period up to 2007.

Horizon:	1	2	3	4	5	6	7	8	9	10	11	12
VECM(HQ-PIC)	1.058	1.056	1.014	1.057	1.051	1.206	1.005	0.886	0.907	0.955	0.908	0.842
	[0.27]	[0.216]	[0.013]	[0.124]	[0.246]	[1.409]	[0.001]	[0.795]	[0.63]	[0.122]	[0.59]	[1.208]
VECM(HQ-J)	0.952	0.955	0.997	0.919	0.949	0.96	0.927	0.892	0.845	0.88	0.86	0.84
	[0.906]	[0.839]	[0.011]	[1.452]	[0.9]	[0.58]	[2.119]	[1.56]	[1.517]	[1.3]	[1.345]	[1.317]
VECM(HQ)Rank	0.952	0.871	0.901	0.987	0.931	0.958	0.955	0.914	0.894	0.886	0.865	0.878
	[0.948]	[1.122]	[1.317]	[0.471]	[0.931]	[0.341]	[1.971]	[1.593]	[1.597]	[1.406]	[1.367]	[1.273]
PV	0.946	0.923	0.932	0.956	0.923	0.896	0.874	0.895	0.804	0.801	0.81	0.801
	[1.372]	[0.843]	[0.965]	[1.131]	[0.836]	[1.191]	[1.514]	[1.196]	[1.549]	[1.328]	[1.235]	[1.248]

Ratio of root-mean-squared-forecast error (RMSFE) of model in each row to that of the VAR in levels, transformed to first differences. Best Models in blue.

*Denotes rejection of the null of equal forecast accuracy at the 10% level, according to the Giacomini and White (2006) test. The number in [] is the test statistic and the critical value is equal to 2.705.

**Denotes rejection of the null of equal forecast accuracy at the 5% level, according to the Giacomini and White (2006) test. The number in [] is the test statistic and the critical value is equal to 3.8415

***Denotes rejection of the null of equal forecast accuracy at the 1% level, according to the Giacomini and White (2006) test. The number in [] is the test statistic and the critical value is equal to 6.6349.

Table 3: Relative RMSFE of restricted models vs VAR for i_{lr}. Forecast period up to 2007.

Horizon:	1	2	3	4	5	6	7	8	9	10	11	12
VECM(HQ-PIC)	0.944	1.06	1.064	1.174	0.993	1.111	0.988	0.914	0.927	0.905	1.022	0.872
	[0.176]	[0.254]	[0.224]	[1.549]	[0.011]	[1.564]	[0.012]	[0.564]	[0.509]	[0.668]	[0.195]	[1.831]
VECM(HQ-J)	0.943	0.939	0.996	0.981	0.905	0.913	0.874	0.944	0.889	0.845	0.874	0.864
	[0.699]	[1.221]	[0.015]	[0.226]	[1.693]	[2.021]	[1.422]	[1.242]	[1.411]	[1.495]	[1.03]	[1.856]
VECM(HQ)Rank	0.915	0.806	0.996	0.924	0.923	0.938	0.859	0.892	0.899	0.831	0.881	0.862
	[0.492]	[1.632]	[0.000]	[1.292]	[2.373]	[1.667]	[1.464]	[1.108]	[1.333]	[1.536]	[1.015]	[1.837]
PV	0.979	0.938	0.963	0.933	0.979	0.933	0.986	0.967	0.881	0.855	0.979	0.809
	[0.15]	[1.051]	[0.587]	[0.715]	[0.161]	[1.807]	[0.039]	[0.237]	[1.045]	[0.936]	[0.067]	[1.733]

See Notes of Table 2.

– see Tables 4 and 5 – the PV model (5) performs well for the short rate, except for the medium horizon, which is dominated by strategy (4, VECM(HQ)Rank). For the long rate, strategy (4, VECM(HQ)Rank) dominates, although the PV model (5) performs well in the medium- to long-horizon. Thus, for interest-rate forecasts, we conclude that the PV model (5) and strategy (4, VECM(HQ)Rank) perform really well. Despite that, it should be noted that no strategy has a forecast performance that is statistically superior to that of the VAR, the exception being strategy (2, VECM(HQ-J)) at the 7-year-ahead horizon.

In Tables 6 through 9, we present forecast results for prices and dividends for the level of the S&P 500 portfolio, P_t and D_t. In forecasting P_t, the PV model (5) performed really well regardless of the forecast sample (2007 or 2012), although

Table 4: Relative RMSFE of restricted models vs VAR for i_{sr}. Forecast period up to 2012.

Horizon:	1	2	3	4	5	6	7	8	9	10	11	12
VECM(HQ-PIC)	1.13	1.043	0.946	1.166	0.955	1.133	1.191	0.919	0.94	1.067	0.862	0.775
	[1.345]	[0.212]	[0.119]	[1.372]	[0.163]	[0.877]	[0.996]	[0.635]	[0.393]	[0.359]	[0.855]	[1.331]
VECM(HQ-J)	1.008	0.944	0.986	0.973	0.939	0.928	0.935	0.94	0.895	0.913	0.866	0.873
	[0.112]	[1.594]	[0.601]	[1.142]	[1.21]	[1.172]	[2.348]	[1.587]	[1.475]	[1.262]	[1.257]	[1.207]
VECM(HQ)Rank	1.038	0.912	0.896	1.003	0.912	0.887	0.885	0.926	0.843	0.895	0.832	0.853
	[0.518]	[0.916]	[1.289]	[0.000]	[1.41]	[1.41]	[1.941]	[1.62]	[1.469]	[1.342]	[1.3]	[1.246]
PV	0.984	0.927	0.922	0.967	0.882	0.876	0.886	0.883	0.805	0.829	0.76	0.758
	[0.303]	[1.119]	[1.209]	[1.243]	[1.64]	[1.394]	[1.626]	[1.292]	[1.504]	[1.349]	[1.32]	[1.292]

See Notes of Table 2.

Table 5: Relative RMSFE of restricted models vs VAR for i_{lr}. Forecast period up to 2012.

Horizon:	1	2	3	4	5	6	7	8	9	10	11	12
VECM(HQ-PIC)	1.105	1.021	1.068	1.118	0.961	1.098	1.234	0.948	0.978	1.05	0.959	0.886
	[0.913]	[0.024]	[0.326]	[1.713]	[0.427]	[1.398]	[1.536]	[0.316]	[0.098]	[0.377]	[0.328]	[2.175]
VECM(HQ-J)	0.971	0.958	1.026**	0.985	0.973	0.958	0.972*	0.984	0.958	0.944	0.928	0.909
	[2.144]	[1.472]	[4.44]	[0.238]	[1.74]	[2.646]	[3.089]	[0.943]	[1.117]	[1.351]	[0.706]	[1.999]
VECM(HQ)Rank	0.927	0.823	1.011	0.95	0.957	0.948	0.94	0.937	0.922	0.918	0.916	0.888
	[0.216]	[1.468]	[0.292]	[1.03]	[2.219]	[2.006]	[2.561]	[0.944]	[1.233]	[1.445]	[0.765]	[1.876]
PV	0.937	0.881	0.965	0.939	0.982	0.97	0.998	0.959	0.899	0.882	0.871	0.849
	[0.512]	[1.616]	[0.611]	[0.716]	[0.003]	[0.004]	[0.315]	[0.255]	[0.845]	[0.892]	[0.752]	[1.853]

See Notes of Table 2.

its performance in the short horizon up to 2007 was beaten by that of strategy (2, VECM(HQ-PIC)) and strategy (3, VECM(HQ-J)). Regarding D_t, for short horizons, strategy (3, VECM(HQ-J)) does well overall. Strategy (2, VECM(HQ-PIC)) does well for the medium horizon, while the PV model (5) performs well in the long horizon. Thus, for P_t and D_t, we conclude that PV model (5) and strategies (2, VECM(HQ-PIC)) and (3, VECM(HQ-J)) are the best.

In Tables 10 through 13, we present forecast results for p_t and d_t. When forecasts

Table 6: Relative RMSFE of restricted models vs VAR for P_t. Forecast period up to 2007.

Horizon:	1	2	3	4	5	6	7	8	9	10	11	12
VECM(HQ-PIC)	0.951	0.951	0.954*	0.959	0.958	0.971	0.99	0.995	0.987	0.994	0.998	0.993
	[0.281]	[2.461]	[3.031]	[2.144]	[2.005]	[1.386]	[1.323]	[1.363]	[1.737]	[0.515]	[0.223]	[1.828]
VECM(HQ-J)	0.945	0.954*	0.962*	0.965	0.96	0.971	0.987	0.991	0.984**	0.992	0.997	0.993
	[0.185]	[3.743]	[3.483]	[2.621]	[2.35]	[1.924]	[1.368]	[1.467]	[6.389]	[0.706]	[0.302]	[1.799]
VECM(HQ)Rank	0.95	0.955	0.958	0.962*	0.958	0.97	0.989	0.994	0.985**	0.992	0.997	0.992
	[0.252]	[1.487]	[1.394]	[2.778]	[2.369]	[0.025]	[0.091]	[0.771]	[4.737]	[0.791]	[0.353]	[1.847]
PV	0.965	0.968	0.983	0.985	0.973	0.982	0.991	0.98	0.974*	0.983	0.989	0.993
	[0.189]	[1.371]	[0.904]	[0.829]	[1.356]	[1.192]	[1.134]	[1.066]	[2.766]	[0.822]	[0.737]	[0.302]

See Notes of Table 2.

Table 7: Relative RMSFE of restricted models vs VAR for D_t. Forecast period up to 2007.

Horizon:	1	2	3	4	5	6	7	8	9	10	11	12
VECM(HQ-PIC)	0.977	0.989	0.901**	0.867***	0.875*	0.902	0.928	0.946	0.949	0.973	0.995	0.973
	[0.927]	[0.623]	[4.317]	[9.852]	[3.5]	[0.085]	[0.034]	[1.51]	[0.000]	[0.448]	[0.034]	[0.285]
VECM(HQ-J)	0.931***	0.887***	0.841***	0.838***	0.895	0.924	0.944	0.941	0.931	0.958	0.982	0.963
	[6.922]	[7.557]	[9.489]	[13.597]	[0.766]	[0.036]	[0.125]	[1.598]	[0.055]	[1.425]	[0.204]	[0.585]
VECM(HQ)Rank	0.958*	0.983	0.869***	0.854***	0.898	0.917	0.935	0.933	0.923	0.955	0.982	0.965
	[3.092]	[2.676]	[7.495]	[9.022]	[0.824]	[0.016]	[0.237]	[1.76]	[0.028]	[1.885]	[0.206]	[0.584]
PV	0.958**	1.107	1.022	1.032	1.267	1.397**	1.467***	1.42	1.375	1.401	1.403	1.337*
	[4.794]	[0.059]	[0.472]	[0.042]	[2.458]	[5.715]	[6.753]	[1.279]	[2.271]	[1.486]	[2.189]	[2.89]

See Notes of Table 2.

Table 8: Relative RMSFE of restricted models vs VAR for P_t. Forecast period up to 2012.

Horizon:	1	2	3	4	5	6	7	8	9	10	11	12
VECM(HQ-PIC)	0.998	0.967**	0.938	0.926	0.898	0.949	0.926	0.918	0.924	0.921	0.888	0.905
	[0.941]	[4.334]	[2.145]	[1.713]	[1.629]	[1.769]	[1.216]	[1.31]	[1.251]	[1.217]	[1.188]	[1.106]
VECM(HQ-J)	0.991	0.97**	0.948	0.935	0.907	0.944	0.926	0.917	0.923	0.921	0.89	0.904
	[1.343]	[4.962]	[2.321]	[1.883]	[1.737]	[1.295]	[1.288]	[1.326]	[1.268]	[1.223]	[1.186]	[1.109]
VECM(HQ)Rank	0.991	0.965	0.936	0.925	0.893	0.941	0.919	0.909	0.915	0.914	0.879	0.895
	[0.544]	[2.667]	[1.962]	[1.91]	[1.665]	[0.822]	[1.354]	[1.322]	[1.242]	[1.217]	[1.185]	[1.107]
PV	0.98	0.908	0.828	0.777	0.727	0.652	0.564	0.478	0.443	0.374	0.315	0.264
	[0.605]	[1.333]	[0.894]	[0.713]	[1.277]	[1.519]	[1.17]	[1.197]	[1.215]	[1.175]	[1.155]	[1.106]

See Notes of Table 2.

Table 9: Relative RMSFE of restricted models vs VAR for D_t. Forecast period up to 2012.

Horizon:	1	2	3	4	5	6	7	8	9	10	11	12
VECM(HQ-PIC)	0.931	0.9	0.871*	0.907**	0.991	1.093	1.108	0.948	0.619	0.632	0.739	0.868
	[0.039]	[2.609]	[2.742]	[4.489]	[0.501]	[0.445]	[0.506]	[0.000]	[1.363]	[1.15]	[1.056]	[0.722]
VECM(HQ-J)	0.872***	0.809***	0.819***	0.891*	1.023	1.114	1.123	0.954	0.618	0.63	0.723	0.868
	[11.218]	[9.108]	[9.954]	[3.088]	[0.084]	[0.502]	[0.58]	[1.017]	[1.378]	[1.156]	[1.06]	[0.74]
VECM(HQ)Rank	0.889*	0.878*	0.863**	0.937	1.06	1.151	1.164	0.989	0.584	0.626	0.76	0.923
	[3.562]	[3.281]	[5.57]	[0.279]	[0.212]	[0.59]	[0.649]	[0.362]	[1.368]	[1.148]	[0.993]	[0.383]
PV	0.888**	0.982	0.971	1.075	1.359**	1.446**	1.422***	1.161	0.843	0.624	0.691	0.833
	[4.878]	[1.198]	[1.406]	[0.567]	[4.698]	[6.42]	[6.763]	[0.249]	[0.492]	[0.795]	[0.674]	[0.445]

See Notes of Table 2.

Table 10: Relative RMSFE of restricted models vs VAR for p_t. Forecast period up to 2007.

Horizon:	1	2	3	4	5	6	7	8	9	10	11	12
VECM(HQ-PIC)	0.929**	0.938**	0.934**	0.947*	0.958	0.965	0.974	0.995	0.993	0.995	0.999	0.997
	[3.985]	[4.604]	[6.117]	[3.034]	[1.007]	[0.803]	[0.719]	[0.211]	[0.149]	[0.118]	[0.018]	[0.019]
VECM(HQ-J)	0.897**	0.921***	0.923***	0.936**	0.935*	0.958	0.97	0.99	0.981	0.987	0.994	0.994
	[6.36]	[6.689]	[8.232]	[5.103]	[3.233]	[1.271]	[0.81]	[0.366]	[0.641]	[0.397]	[0.126]	[0.101]
VECM(HQ)Rank	0.901**	0.931**	0.929**	0.942*	0.935*	0.959	0.972	0.991	0.979	0.984	0.992	0.995
	[4.958]	[4.593]	[5.979]	[3.393]	[3.102]	[1.214]	[0.812]	[0.357]	[0.668]	[0.487]	[0.171]	[0.051]
PV	2.025***	2.917***	3.119***	3.178***	3.176***	3.286***	3.407***	3.352***	3.325***	3.328***	3.335***	3.345***
	[42.408]	[79.156]	[67.565]	[58.132]	[48.619]	[43.036]	[37.053]	[32.576]	[28.27]	[25.879]	[23.389]	[21.084]

See Notes of Table 2.

Table 11: Relative RMSFE of restricted models vs VAR for d_t. Forecast period up to 2007.

Horizon:	1	2	3	4	5	6	7	8	9	10	11	12
VECM(HQ-PIC)	0.98	0.964	0.911	0.891	0.911	0.937	0.952	0.966	0.971	0.977	0.968	0.949
	[2.433]	[1.977]	[1.995]	[0.998]	[0.761]	[0.571]	[0.718]	[0.992]	[0.908]	[0.475]	[0.338]	[0.474]
VECM(HQ-J)	0.952	0.873**	0.806***	0.826**	0.928	0.958	0.965	0.941	0.917	0.937	0.939	0.93
	[2.304]	[5.904]	[7.568]	[5.988]	[0.99]	[0.477]	[0.604]	[2.013]	[2.646]	[1.434]	[1.022]	[1.026]
VECM(HQ)Rank	0.964	0.94	0.812	0.825**	0.919	0.945	0.955	0.934	0.912*	0.929	0.938	0.934
	[0.234]	[0.002]	[2.574]	[4.742]	[1.618]	[0.755]	[0.706]	[2.188]	[2.952]	[1.762]	[1.161]	[1.022]
PV	0.907	3.324	5.371**	5.929**	5.909***	6.016***	6.221***	6.355***	7.327***	8.269***	8.408***	8.308***
	[1.742]	[2.356]	[5.903]	[4.93]	[9.502]	[9.275]	[10.018]	[8.809]	[9.065]	[10.561]	[10.227]	[10.967]

See Notes of Table 2.

are carried out until 2007, the best model for p_t is strategy (3, VECM(HQ-J)), while there is no clear best strategy for d_t: for short horizons the log PV model (6) performs well, for the medium horizons strategy (3, VECM(HQ-J)) is best, while for long horizons strategy (4, VECM(HQ)Rank) is the best one. For the full forecast sample up to 2012, the best model for p_t is strategy (3, VECM(HQ-J)), while for d_t log PV model (6) performs well for $h = 1$, strategy (3, VECM(HQ-J)) is best for $h = 2, 3$, while for long horizons strategy (4, VECM(HQ)Rank) is the best one. Thus, we conclude that strategy (3, VECM(HQ-J)) works really well, followed by strategy (4, VECM(HQ)Rank). On occasion, log PV model (6) performs well.

Wrapping up results across all assets and horizons, we find that PV models (especially 5) perform well. The same can be said about strategy (3, VECM(HQ-J)) and strategy (4, VECM(HQ)Rank). However, as is well known, econometric models are built to forecast mainly at shorter horizons, since, as the horizon increases, conditional forecasts converge to their unconditional counterparts. If we focus only on

Table 12: Relative RMSFE of restricted models vs VAR for p_t. Forecast period up to 2012.

Horizon:	1	2	3	4	5	6	7	8	9	10	11	12
VECM(HQ-PIC)	0.868*	0.909	0.945	0.962	0.969	0.984	0.981	0.988	1.002	1.001	1.000	1.003
	[3.641]	[2.255]	[0.932]	[1.489]	[0.803]	[0.1]	[0.091]	[0.103]	[0.054]	[0.003]	[0.002]	[0.025]
VECM(HQ-J)	0.876**	0.901*	0.924**	0.943*	0.958	0.975	0.976	0.984	0.989	0.995	0.999	1.001
	[5.729]	[3.358]	[4.425]	[2.949]	[1.599]	[0.42]	[0.322]	[0.203]	[0.553]	[0.09]	[0.001]	[0.001]
VECM(HQ)Rank	0.879**	0.908*	0.936	0.949	0.957	0.976	0.98	0.986	0.991	0.998	1.001	1.000
	[6.201]	[2.751]	[2.425]	[1.906]	[1.369]	[0.364]	[0.145]	[0.108]	[0.449]	[0.032]	[0.004]	[0.000]
PV	2.001***	2.834***	3.044***	3.104***	3.14***	3.204***	3.212***	3.19***	2.948***	3.027***	3.037***	3.051***
	[32.221]	[81.764]	[56.782]	[48.517]	[38.35]	[36.047]	[34.488]	[33.279]	[32.957]	[33.91]	[32.3]	[37.36]

See Notes of Table 2.

Table 13: Relative RMSFE of restricted models vs VAR for d_t. Forecast period up to 2012.

Horizon:	1	2	3	4	5	6	7	8	9	10	11	12
VECM(HQ-PIC)	0.961	0.966	0.929	0.893	0.86	0.873	0.88	0.885	0.942	0.969	0.991	0.991
	[0.069]	[0.363]	[0.387]	[1.373]	[0.212]	[0.61]	[1.491]	[2.175]	[2.097]	[0.963]	[0.157]	[0.071]
VECM(HQ-J)	0.921**	0.889*	0.826**	0.809**	0.869	0.86	0.851	0.841	0.928*	0.96	0.977	0.976
	[4.918]	[2.804]	[4.763]	[4.612]	[1.375]	[1.124]	[1.438]	[2.494]	[2.875]	[1.666]	[0.774]	[0.641]
VECM(HQ)Rank	0.933**	0.976	0.865	0.807*	0.844	0.848	0.859	0.852	0.924*	0.952	0.974	0.973
	[4.472]	[0.679]	[2.077]	[3.544]	[1.975]	[1.899]	[1.671]	[2.515]	[3.122]	[1.942]	[0.816]	[0.823]
PV	0.819***	2.777***	4.943***	6.345***	6.77***	6.801***	6.864***	7.005***	5.678***	5.863**	5.779**	5.538**
	[7.448]	[18.576]	[24.418]	[20.669]	[14.578]	[9.772]	[8.519]	[8.137]	[7.423]	[6.5]	[5.757]	[5.273]

See Notes of Table 2.

results at business-cycle horizons (1 through 5 years ahead), we have a clear winner strategy – strategy (3, VECM(HQ-J)) – which produces the best forecast model for 36.67% of occasions. The other two strategies, strategy (4, VECM(HQ)Rank) and PV model, produce the best models for 26.67% and 25% of occasions, respectively.

Finally, we investigate if one of the 3 preferred methods – strategy (3, VECM(HQ-J)), strategy (4, VECM(HQ)Rank), and the PV model – produce forecasts that are statistically different from the others, employing the unconditional predictive ability test of Giacomini and White (2006). For i_{lr} and i_{sr}, we find that strategy (3, VECM(HQ-J)) have produced statistically better accuracy statistics than the PV model, while the converse is not true. Regarding the latter and strategy (4, VECM(HQ)Rank), there is no clear dominance, which also happens when we compared strategy (3, VECM(HQ-J)) and strategy (4, VECM(HQ)Rank). For P_t and D_t, we find that strategy (3, VECM(HQ-J)) have produced statistically better accuracy statistics than the PV model and strategy (4, VECM(HQ)Rank), respectively, while the converse is not true. For p_t and d_t, there is a clear pecking order as follows: strategy (3, VECM(HQ-J)), strategy (4, VECM(HQ)Rank), and the PV model.

All in all, if we have to recommend one forecast strategy for multivariate models containing series subject to PVM restrictions, we would choose strategy (3, VECM(HQ-J)). Notice that it imposes cointegration restriction (theory) but estimates the cointegrating vector using econometric techniques applied to data.

5 Conclusion

This paper has two original contributions. The first is to show that PV relationships entail a weak-form SCCF restriction, as in Hecq et al. (2006) and in Athanasopoulos et al. (2011). It also implies a polynomial serial correlation common feature relationship (Cubadda and Hecq, 2001) for the VAR representation for Δy_t and the cointegrating relationship $Y_t - \theta y_t$. These represent short-run restrictions on the dynamic system for these variables, something that has not been discussed before.

Our second contribution relates to forecasting multivariate time series that are subject to PVM restrictions, which has a wide application in macroeconomics and finance. We benefit from previous work showing the benefits for forecasting when the short-run dynamics of the system is constrained for stationary data (Vahid and Issler, 2002), and when it is constrained for data subject to long- and short-run restrictions (Issler and Vahid, 2001, Anderson and Vahid, 2011, and Athanasopoulos et al., 2011). The reason why appropriate common-cycle restrictions improve forecasting is because it finds linear combinations of the first differences of the data that cannot be forecast by past information. This embeds natural exclusion restrictions preventing the estimation of useless parameters, which would otherwise contribute to the increase of forecast variance with no expected reduction in bias.

We applied the techniques discussed in this paper to data known to be subject to PV restrictions: the online series maintained and updated by Shiller at http://www.econ.yale.edu/~shiller/data.htm. We focus on three different data sets. The first includes the levels of interest rates with long and short maturities, the second includes the level of real price and dividend for the S&P composite index, and the third includes the logarithmic transformation of prices and dividends, which PV analysis requires the inclusion of the real risk-free rate. Our exhaustive investigation of six different multivariate models reveals that better forecasts can be achieved when restrictions are applied to them. Specifically, cointegration restrictions in strategy (3, VECM(HQ-J)) and cointegration and weak-form SCCF restrictions in strategy (4, VECM(HQ)Rank), as well as all the set of theoretical restrictions embedded in the PV model (5). All in all, if we have to recommend one forecast strategy for multivariate models containing series subject to PVM restrictions, we would choose strategy (3, VECM(HQ-J)).

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A Appendix: Testing present-value models with a common-cycle approach

The discussion of this paper suggests that, for integrated Y_t and y_t, there are three different instances in which we can investigate the validity of PVMs. First, the cointegration test for Y_t and y_t, if both are $I(1)$. Second, the (invariant) rank restrictions in the VECM or the transformed VAR. Third, the coefficient restrictions and unpredictability properties for linear combinations. In order to test for PVMs, we propose the following steps:

1. Choose consistently the order of the $VAR(p)$ for the joint $I(1)$ process $(Y_t, y_t)'$ using different information criteria.

2. Given our choice of p, test for the existence of cointegration between Y_t and y_t. If that is the case (there exists one cointegrating vector), estimate the long-run coefficient θ, in $S_t = Y_t - \theta y_t$, super-consistently using the likelihood-based trace test proposed by Johansen (1995). Alternatively, the Engle and Granger (1987) regression test can be carried out. In either case, form $\hat{S}_t = Y_t - \hat{\theta} y_t$. If there is no cointegration, the PVM is rejected.

3. Given p and $\hat{S}_t$, test for the weak form common feature using a reduced rank test for $(\Delta Y_t, \Delta y_t)'$. We can use both a likelihood ratio multivariate approaches (e.g. a canonical correlation analysis) and a single-equation approach (e.g. GMM). Because most present-value relationships apply to heteroskedastic financial data, one may prefer a GMM framework on the basis that it easily embeds robust variance-covariance matrices for parameters estimates. Indeed the canonical correlation approach assumes *i.i.d.* disturbances.

   Note that we can improve over steps 1 to 3 using steps 4 and/or 5 below. Given that we only work with bivariate systems for a relatively large number of observations in this paper we do not introduce those small sample improvements into our analysis. But, these are:

4. Integrate steps 2 and 3, estimating jointly long-run and short-run parameters as in Centoni, Cubadda and Hecq (2007).

Integrate steps 1, 2 and 3, estimating jointly the lag length of the VAR and long-run and short-run parameters as in Athanasopoulos *et al.* (2011).

A.1 LR tests for i.i.d. disturbances

The canonical-correlation approach entails the use of a likelihood ratio (reduced-rank regression) test for the weak-form common features in the $VECM\,(p-1)$ for $(\Delta Y_t, \Delta y_t)'$. It can be undertaken using the canonical-correlation test on zero eigenvalues, which are computed from:

$$CanCor\left\{\begin{pmatrix}\Delta Y_t\\ \Delta y_t\end{pmatrix}, \begin{pmatrix}\Delta Y_{t-1}\\ \vdots\\ \Delta Y_{t-p+1}\\ \Delta y_{t-1}\\ \vdots\\ \Delta y_{t-p+1}\end{pmatrix} \mid (D_t, \hat{S}_{t-1})\right\}, \tag{38}$$

where $CanCor\,\{X_t, W_t|G_t\}$ denotes the computation of canonical correlations between the two sets of variables X_t and W_t, concentrating out the effect of G_t (deterministic terms and a disequilibrium error-correction term) by multivariate least squares. The previous program (38) is numerically equivalent to

$$CanCor\left\{\begin{pmatrix}\Delta Y_t\\ \Delta y_t\\ \hat{S}_{t-1}\end{pmatrix}, \begin{pmatrix}\Delta Y_{t-1}\\ \vdots\\ \Delta Y_{t-p+1}\\ \Delta y_{t-1}\\ \vdots\\ \Delta y_{t-p+1}\\ \hat{S}_{t-1}\end{pmatrix} \mid D_t\right\} \tag{39}$$

which is more convenient to directly obtain the coefficient of $\hat{S}_{t-1}$ in (10). The likelihood ratio test, denoted by ξ_{LR}, considers the null hypothesis that there exist at least s common feature vectors. It is obtained in $\xi_{LR} = -T\sum_{i=1}^{s}\ln(1-\hat{\lambda}_i), \qquad s = 1, 2,$ where $\hat{\lambda}_i$ are the i-th smallest squared canonical correlations computed from (38) or (39) above, namely from $\hat{\Sigma}_{XX}^{-1}\hat{\Sigma}_{XW}\hat{\Sigma}_{WW}^{-1}\hat{\Sigma}_{WX}$, or similarly from the symmetric matrix $\hat{\Sigma}_{XX}^{-1/2}\hat{\Sigma}_{XW}\hat{\Sigma}_{WW}^{-1}\hat{\Sigma}_{WX}\hat{\Sigma}_{XX}^{-1/2}$, where $\hat{\Sigma}_{ij}$ are the empirical covariance matrices, $i, j = X, W$.

In the bivariate case, the unrestricted VECM has $4(p-1)+2$ parameters, whereas the restricted model has $2(p-1)+2+1$. The number of restrictions when testing the hypothesis that there exists one WF common feature is then $2(p-1)-1 = 2p-3$ for $p > 1$.[7] As proposed in Issler and Vahid (2001), we can obtain the same statistics by computing twice the difference between the log-likelihood in the unrestricted $VECM\,(p-1)$ for $(\Delta Y_t, \Delta y_t)'$ and in the pseudo-structural form estimated by FIML:

$$\begin{pmatrix} 1 & -\gamma_0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \Delta Y_t \\ \Delta y_t \end{pmatrix} = \begin{pmatrix} 0 \\ \tilde{\Gamma}_1 \end{pmatrix}\begin{pmatrix} \Delta Y_{t-1} \\ \Delta y_{t-1} \end{pmatrix} + \ldots$$

$$+\begin{pmatrix} 0 \\ \tilde{\Gamma}_{p-1} \end{pmatrix}\begin{pmatrix} \Delta Y_{t-p+1} \\ \Delta y_{t-p+1} \end{pmatrix} + \begin{pmatrix} (\alpha_1 - \gamma_0\alpha_2) \\ \tilde{\alpha}_2 \end{pmatrix} S_{t-1} + \begin{pmatrix} v_{1t} \\ v_{2t} \end{pmatrix}.$$

For the transformed VAR the restriction underlying the restricted PSCCF can be tested using:

$$CanCor\left\{\begin{pmatrix} \hat{S}_t \\ \Delta y_t \\ \hat{S}_{t-1} \end{pmatrix}, \begin{pmatrix} \hat{S}_{t-1} \\ \hat{S}_{t-2} \\ \vdots \\ \hat{S}_{t-p} \\ \Delta y_{t-2} \\ \vdots \\ \Delta y_{t-p+1} \end{pmatrix} \mid D_t \right\},$$

where the number of parameters in the unrestricted model is $4(p-1)+2$; the restricted model has $6+2(p-2)$, the number of restrictions is $2p-4$ in case of

[7] In the VECM, the general formula for n series that can be annihilated by s combinations is $sn(p-1) - s(n-s)$.

unrestricted $\tilde{\gamma}_1$:

$$\begin{pmatrix} 1 & -\tilde{\gamma}_0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} S_t \\ \Delta y_t \end{pmatrix} = \begin{pmatrix} \tilde{\Gamma}_{1a} \\ \tilde{\Gamma}_{1b} \end{pmatrix} \begin{pmatrix} S_{t-1} \\ \Delta y_{t-1} \end{pmatrix} + ...$$

$$+ \begin{pmatrix} 0 \\ \tilde{\Gamma}_{p-1} \end{pmatrix} \begin{pmatrix} S_{t-p+1} \\ \Delta y_{t-p+1} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ \tilde{\Gamma}_{p,p} & 0 \end{pmatrix} \begin{pmatrix} S_{t-p} \\ \Delta y_{t-p} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}.$$

If $\tilde{\gamma}_1$ is restricted we have $2p-3$ restrictions and the pseudo structural form is

$$\begin{pmatrix} 1 & -\tilde{\gamma}_0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} S_t \\ \Delta y_t \end{pmatrix} = \begin{pmatrix} \vartheta_1 & 0 \\ \tilde{\Gamma}_{2,1} & \tilde{\Gamma}_{2,2} \end{pmatrix} \begin{pmatrix} S_{t-1} \\ \Delta y_{t-1} \end{pmatrix} +$$

$$\begin{pmatrix} 0 \\ \tilde{\Gamma}_{p-1} \end{pmatrix} \begin{pmatrix} S_{t-2} \\ \Delta y_{t-2} \end{pmatrix} + \cdots + \begin{pmatrix} 0 & 0 \\ \tilde{\Gamma}_{p,p} & 0 \end{pmatrix} \begin{pmatrix} S_{t-p} \\ \Delta y_{t-p} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}.$$

Notice that this set of rank restrictions are identical to the ones in Campbell and Shiller (1987) if one imposes zero restrictions in the last matrix coefficient in their setup[8]. The proposed approach to testing PVMs here is to first test the rank condition (necessary) without imposing yet any further parameter restrictions. As argued above, the rank condition is invariant to how we write the PV equation linking Y_t and y_t. If not rejected, then we can test the additional restrictions on matrices coefficients, which are not invariant to how we write the PV equation. Putting more weight on invariant restrictions satisfies robustness, since, not only a different definition of the timing of Y_t and/or y_t, but also the presence of measurement error, data revisions, all will lead to the correct rank condition to be met but imply different parameter values in the difference equation generating PVMs. An additional reason to follow this path is that we will be able to split both effects, shedding light on the

[8] This is probably implicit in their analysis but it is not discussed in the paper itself.

exact reason for rejecting theory if that is the case. Understanding why we reject a given PVM is an important issue, since different authors have complained that cross-equation restriction tests reject PVMs too often, even in cases where theory is firmly believed to hold and that graphical analysis seems to support that view.

A.2 Regression-Based GMM Tests

Testing with a GMM approach entails testing the common feature null hypothesis using an orthogonality condition between a combination of variables in the model $\left(\Delta Y_t, \Delta y_t, \hat{S}_{t-1}\right)'$ and the conditioning set W_t'. For example, in the context of (8), we would have the following moment restrictions:

$$\mathbb{E}([\Delta Y_t - \gamma_1 \Delta y_t - \gamma_2 \hat{S}_{t-1}] \otimes W_t') = 0, \tag{40}$$

where we would have additionally to test H_0: $\gamma_1 = 0$ and $\gamma_2 = \frac{1-\delta}{\delta}$ using a Wald test. Prior to that, we want to estimate γ_1 and γ_2 and test the validity of the over-identifying restrictions in (40). The use of IV type estimators and the associated orthogonality tests is straightforward in this context. Let us consider W_t the vector of instruments defined as before (an intercept is added). The GIVE estimator is simply the 2SLS or the IV estimator when the instruments are the past of the series, namely

$$\hat{\theta}_{GIVE} = \left(\mathbf{\Delta X}'\mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'\mathbf{\Delta X}\right)^{-1}\left(\mathbf{\Delta X}'\mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'\mathbf{\Delta Y}\right), \tag{41}$$

with $\mathbf{\Delta X}_t$= $(\Delta y_t, \hat{S}_{t-1}, 1)'$. The validity of the orthogonality condition and consequently the presence of a common feature vector is obtained via an overidentification

J-test (Hansen, 1982) $J_1(\theta) = Tg_T(\theta;.)'P_T^{-1}g_T(\theta;.)$,whose empirical counterpart is:

$$J_1(\theta_{IV}) = (\mathbf{u}'\tilde{\mathbf{W}})(\hat{\sigma}_u^2\tilde{\mathbf{W}}'\tilde{\mathbf{W}})^{-1}(\tilde{\mathbf{W}}'\mathbf{u}).$$

The variance-covariance matrix of the orthogonality condition has under usual regularity properties the sample counterpart $\hat{P}_T = (1/T)\hat{\sigma}_u^2(\tilde{\mathbf{W}}'\tilde{\mathbf{W}})$ with $u_t = \Delta Y_t - \hat{\gamma}_1\Delta y_t - \hat{\gamma}_2\hat{S}_{t-1}$. $\tilde{\mathbf{W}}$ is the demeaned $\mathbf{W}$, namely $\tilde{\mathbf{W}} = \mathbf{W} - \mathbf{i}(\mathbf{i}'\mathbf{i})^{-1}\mathbf{i}\mathbf{W}$ (with $\mathbf{i} = (1...1)'$) because we do not want to impose that the common feature vector also annihilates the constant vector.

All the estimates and tests presented above embedded the assumption of homoskedasticity. This may be fine for macroeconomic data, such as consumption and income, but is clearly at odds with financial data. Candelon *et al.* (2005) have illustrated in a Monte Carlo exercise that ξ_{LR} has large size distortions in the presence of GARCH disturbances. We implement the GIVE estimator by using the White's HCSE estimator such that (see Hamilton, 1994):

$$\hat{\theta}_{GMM} = \left(\mathbf{\Delta X}'\mathbf{W}(\mathbf{W}'\mathbf{B}\mathbf{W})^{-1}\mathbf{W}'\mathbf{\Delta X}\right)^{-1}\left(\mathbf{\Delta X}'\mathbf{W}(\mathbf{W}'\mathbf{B}\mathbf{W})^{-1}\mathbf{W}'\mathbf{\Delta Y}\right), \qquad (42)$$

where the only difference between $\hat{\theta}_{GMM}$ and the usual $\hat{\theta}_{GIVE}$ is the presence of an additional diagonal matrix $\mathbf{B} = \mathbf{diag}(u_1^2, u_2^2, ...u_T^2)$ where $u_t = \Delta Y_t - \hat{\gamma}_{1IV}\Delta y_t - \hat{\gamma}_{2IV}\hat{S}_{t-1}$ are the residuals obtained under homoskedasticity using the GIVE estimation in a first step. For testing, we form the following new sequence of residuals $u_t^* = \Delta Y_t - \hat{\gamma}_{1GMM}\Delta y_t - \hat{\gamma}_{2GMM}\hat{S}_{t-1}$,and use these to compute a new J-test robust to heteroskedasticity:

$$J_2(\theta_{GMM}) = (\mathbf{u}^{*\prime}\tilde{\mathbf{W}})(\tilde{\mathbf{W}}'\mathbf{B}\tilde{\mathbf{W}})^{-1}(\tilde{\mathbf{W}}'\mathbf{u}^*). \qquad (43)$$

Note that alternative approaches have been evaluated in Hecq an Issler (2012).

A.3 Small sample properties of PVM tests

A small Monte Carlo simulation might help to advise the use of one of the tests considered. We use $T = 100$ observations with $10,000$ replications. The lag length of the VAR in level in the data generating process is chosen to be $p = 3$. However, we estimate the model for $p = 2, 3$ and 5. The DGP that ensures $\gamma' = (1 : 0)$ is:

$$\begin{pmatrix} \Delta Y_t \\ \Delta y_t \end{pmatrix} = \begin{pmatrix} 0.05 \\ 0.05 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0.5 & 0.2 \end{pmatrix} \begin{pmatrix} \Delta Y_{t-1} \\ \Delta y_{t-1} \end{pmatrix} + .. \begin{pmatrix} 0 & 0 \\ -0.4 & 0.2 \end{pmatrix} \begin{pmatrix} \Delta Y_{t-2} \\ \Delta y_{t-2} \end{pmatrix}$$
$$+ \begin{pmatrix} 1 \\ 0.75 \end{pmatrix} \begin{pmatrix} 1 & -\frac{\delta}{1-\delta} \end{pmatrix} \begin{pmatrix} Y_{t-1} \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}.$$

We considered two types of error terms for the VECM above: in the first DGP, labelled DGP # 1 in Table13, the disturbance term is bivariate normal with a unit variance and a correlation of 0.5; in the second process, labelled DGP # 2, the disturbance terms are governed by a bivariate GARCH process (CCC) with a yesterday news coefficient of 0.25, a coefficient of persistence of 0.74, and a long run variance equals 0.01. Note that yesterday's news coefficient is larger than what is usually found empirically (between 0.10 and 0.15). The theoretical coefficients in the relationship $\Delta Y_t = -\gamma_1 \Delta y_t + \gamma_2 S_{t-1} + u_t$ are $\gamma_1 = 0$ and $\gamma_2 = \frac{\delta}{1-\delta}$. Here, for simplicity, we set $\frac{\delta}{1-\delta} = 1$ in the DGP but the cointegrating vector $Y_t - \hat{\theta} y_t$ is estimated using Johansen's approach.

Table 14 reports the empirical rejection frequency at the 5% significance level (nominal size). In the *iid* case, the behavior of the tests is rather similar. Results get much more worse in the presence of time varying conditional variances. With

Table 14: Empirical size (nominal set to 5statistic

		$T = 100$					
			Levels			Log Levels	
	$VAR(p)$	ζ_{LR}	J_1	J_2	ζ_{LR}	J_1	J_2
DGP #1: iid	$p = 2$	6.27	5.96	5.95	6.25	5.99	5.98
	$p = 3$	7.91	6.93	6.17	7.89	6.93	6.18
	$p = 5$	8.62	6.47	5.00	8.65	6.45	4.98
DGP #2:GARCH	$p = 2$	13.2	12.9	6.07	13.3	13.0	6.05
	$p = 3$	17.6	16.5	5.76	17.6	16.5	5.74
	$p = 5$	21.3	17.8	4.53	21.3	17.8	4.54

heteroskedastic data, the robust-White GMM test has proper size.

In the last three columns of Table 14 we report the rejection frequencies of the same three tests applied to the logarithms of the variables Y_t and y_t but for the same DGP in levels. It emerges however that taking the logs does not affect the rejection frequencies.

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