ISSN 1518-3548

BANCO CENTRAL DO BRASIL

Working Paper Series 60

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December, 2002

ISSN 1518-3548
CGC 00.038.166/0001-05

Working Paper Series	Brasília	n. 60	Dec	2002	p. 1 – 22

Working Paper Series

Edited by:

Research Department (Depep)

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Authorized by Ilan Goldfajn (Deputy Governor for Economic Policy).

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Delegated Portfolio Management

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Abstract

In this paper, we examine optimal portfolio decisions within a decentralized framework. There are many portfolio managers choosing optimal portfolio weights in a mean-variance framework and taking decisions in a decentralized way. However, the overall portfolio may not be efficient, as the portfolio managers do not take into account the overall covariance matrix. We show that the initial endowment that portfolio managers can use within the firm in order to manage their portfolios can be used as a control variable by the top administration and redistributed within the firm in order to achieve overall efficiency.

Keywords: optimal portfolio; decentralized; efficient frontier; portfolio management.

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1. Introduction

One of the main tasks of portfolio managers is to achieve the best possible trade-off between risk and return. This task involves the determination of the best risk-return opportunities available from feasible portfolios and the choice of the best portfolio from this feasible set. However, in many situations, the portfolio is managed in a decentralized way, with the top administration of the firm delegating to different portfolio managers partial control about investment in subclasses of assets.

In general, the top administration of trading firms decides the amount of wealth invested in certain asset classes. The top administration however delegates to portfolio managers the control of which specific securities to hold and the proportions of these securities in the portfolio.[1]

The delegation process may imply in an overall inefficient portfolio. Even if all portfolio managers are in the efficient frontier for their particular asset class it may be the case that, in the aggregate, the resulting portfolio is an inefficient one as portfolio managers are not taking into account the overall covariance matrix and all possible combinations that would result in the best trade-off between risk and return.

These considerations has led us to examine which conditions should be met so that even in a decentralized decision making set the overall portfolio would be efficient. Our findings suggest that the initial endowment available for investing by the portfolio managers and the risk free interest rate they face when taking investment decisions can be used as control variables to attain overall efficiency.

Although we recognize that asymmetry of information play a crucial role in the delegation process, in this paper, we put these considerations to concentrate in the pure compatibility of solutions between the different portfolio managers with the overall

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[1] Classical articles in asset allocation choice are Tobin (1958) and Markowitz (1952, 1959).

solution. Bhattacharya and Pfleiderer (1985) develop a model of decentralized investment management under asymmetric information, where they are especially concerned about screening agents with superior information and on the surplus extraction from the agent. Barry and Starks (1984) show that risk sharing considerations are sufficient to motivate the use of multiple managers. Stoughton (1993) investigate the significance of nonlinear contracts on the incentive for portfolio managers to collect information, justifying employment of portfolio managers with the notion of superior skills at acquiring and interpreting information related to movement in security prices.

We solve the problem of finding equivalence between decentralized and centralized portfolio management analytically within a mean-variance framework, which provides some insights on how to enhance the delegation process.[2] Our findings suggest that it is possible to decentralize investment decisions and construct portfolios that are still optimal in an aggregate sense. All that is needed are some instruments to control decisions taken by portfolio managers such as their available initial endowment and the risk free interest rate that they face, which may be determined endogenously in our model.

We solve a general case with many portfolio managers that are specialized in some particular asset class (which may intersect or not in some cases), and show that the initial endowment that portfolio managers have to invest in their portfolios can be used by the top administration to achieve overall efficiency.

The organization of the paper is as follows. In section II, we develop the model. Section III gives some interpretations of the results. Section IV concludes and gives directions for further research.

2 There may be other difficulties in implementing mean-variance analysis such as the extreme weights that may arise when sample efficient portfolios are constructed. As Genotte (1986) and Britten-Jones (1999) noticed, there are huge estimation errors in expected returns estimation, which cannot be ignored. However, we will not address these points in this paper.

2. The model

We start with n risky assets. Let $\mathbf{w}$ be the n x 1 vector of portfolio weights for risky assets, $\mathbf{V}$ the n x n covariance matrix, $\mathbf{r}$ the n x 1 expected return vector for all risky assets, r_f the risk free interest rate and r_p the portfolio's expected rate of return.

We characterize an investor's preferences by utility curves of the following general form:

$$U\left(r_p,\sigma_p\right)=r_p-\lambda\frac{1}{2}\sigma_p^2=\mathbf{w}^{\mathrm{T}}\mathbf{r}+\left(1-\mathbf{w}^{\mathrm{T}}\mathbf{1}\right)r_f-\lambda\frac{1}{2}\mathbf{w}^{\mathrm{T}}\mathbf{V}\mathbf{w}\,. \tag{1}$$

The first part of this utility function is the expected return of one dollar invested in the portfolio and the second part is half of the variance of one dollar invested in the portfolio multiplied by a scalar λ. The convenience of this utility function is that maximizing it is equivalent to find a frontier portfolio, as we will show below. The coefficient λ may be interpreted as a coefficient of risk aversion.[3] Higher levels for $U(\cdot,\cdot)$ imply in higher utility for managers and (1) shows that utility increases if the expected return increases, and it decreases when the variance increases. The relative magnitude of these changes is governed by λ.

An efficient portfolio is one that maximizes expected return for a given level of variance (when there is a solution to this problem). We can represent this problem as:

$$\begin{gathered}\max_{\{\mathrm{w}\}}\ \ \mathbf{w}^{\mathrm{T}}\mathbf{r}+\left(1-\mathbf{w}^{\mathrm{T}}\mathbf{1}\right)r_f\\ \text{s.t.}\\ \frac{1}{2}\mathbf{w}^{\mathrm{T}}\mathbf{\Sigma}\mathbf{w}=\sigma_p^2\ \ .\end{gathered} \tag{2}$$

The Lagrangean for this problem is:

$$\ell=\mathbf{w}^{\mathrm{T}}\mathbf{r}+\left(1-\mathbf{w}^{\mathrm{T}}\mathbf{1}\right)r_f-\lambda\left(\frac{1}{2}\mathbf{w}^{\mathrm{T}}\mathbf{V}\mathbf{w}-\sigma_p^2\right). \tag{3}$$

3 However, this is not the absolute risk aversion as defined in Arrow (1970) which is given by $-U\,''(\cdot)/U\,'(\cdot)$.

where λ is a positive constant. The first order conditions are:

$$\frac{\partial \ell}{\partial \mathbf{w}} = \mathbf{r} - \mathbf{1} r_f - \lambda \mathbf{V}\,\mathbf{w}_p = \mathbf{0}, \tag{4}$$

$$\frac{\partial \ell}{\partial \lambda} = r_p - \mathbf{w}_p^{\mathbf{T}} \mathbf{r} - \left(1 - \mathbf{w}^{\mathbf{T}} \mathbf{1}\right) r_f = 0. \tag{5}$$

Observe that (3) is essentially equivalent to (1). The first order condition of (1) is exactly (4). As $\mathbf{V}$ is a positive definite matrix, it follows that the first order conditions are necessary and sufficient for a global optimum. In this case, we can solve for the portfolio weights:

$$\mathbf{w}_p = \frac{1}{\lambda} \mathbf{V}^{-1} \left(\mathbf{r} - \mathbf{1} r_f\right). \tag{6}$$

In what follows, we will show equation (6) in a more convenient way. If we have n risky assets, the covariance matrix is:

$$\mathbf{V} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdot & \cdot & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \cdot & \cdot & \sigma_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \sigma_{(n-1)n} \\ \sigma_{n1} & \cdot & \cdot & \sigma_{n(n-1)} & \sigma_n^2 \end{bmatrix}.$$

Using Stevens' (1998) direct characterization of the inverse of the covariance matrix, we have that the inverse of the covariance matrix is given by:

$$\mathbf{V}^{-1} = \begin{bmatrix} \frac{1}{\sigma_1^2\left(1-R_1^2\right)} & -\frac{\beta_{12}}{\sigma_1^2\left(1-R_1^2\right)} & \cdot & \cdot & -\frac{\beta_{1n}}{\sigma_1^2\left(1-R_1^2\right)} \\ -\frac{\beta_{21}}{\sigma_2^2\left(1-R_2^2\right)} & \frac{1}{\sigma_2^2\left(1-R_2^2\right)} & \cdot & \cdot & -\frac{\beta_{2N}}{\sigma_2^2\left(1-R_2^2\right)} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ -\frac{\beta_{n1}}{\sigma_n^2\left(1-R_n^2\right)} & -\frac{\beta_{n2}}{\sigma_n^2\left(1-R_n^2\right)} & \cdot & \cdot & \frac{1}{\sigma_n^2\left(1-R_n^2\right)} \end{bmatrix}. \tag{7}$$

where R_i^2 and β_{ij} are the R-squared and the coefficient for the multiple regression of the excess return for the *k*-th asset on the excess returns of all the other assets. The factor $\sigma_1^2\left(1-R_1^2\right)$ is the part of the variance of the excess return for the *k*-th asset that cannot be explained by the regression on the other risky excess return returns, which is equivalent to the estimate of the variance of the residual of that regression.

Using this result, we can rewrite (6) as:

$$\begin{pmatrix} w_1 \\ w_2 \\ \cdot \\ \cdot \\ w_n \end{pmatrix} = \frac{1}{\lambda}\begin{bmatrix} \frac{1}{\sigma_1^2\left(1-R_1^2\right)} & -\frac{\beta_{12}}{\sigma_1^2\left(1-R_1^2\right)} & \cdot & \cdot & -\frac{\beta_{1n}}{\sigma_1^2\left(1-R_1^2\right)} \\ -\frac{\beta_{21}}{\sigma_2^2\left(1-R_2^2\right)} & \frac{1}{\sigma_2^2\left(1-R_2^2\right)} & \cdot & \cdot & -\frac{\beta_{2N}}{\sigma_2^2\left(1-R_2^2\right)} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ -\frac{\beta_{n1}}{\sigma_n^2\left(1-R_n^2\right)} & -\frac{\beta_{n2}}{\sigma_n^2\left(1-R_n^2\right)} & \cdot & \cdot & \frac{1}{\sigma_n^2\left(1-R_n^2\right)} \end{bmatrix}\begin{pmatrix} r_1-r_f \\ r_2-r_f \\ \cdot \\ \cdot \\ r_n-r_f \end{pmatrix}. \tag{8}$$

Letting $e_i = r_i - r_f$ be the excess expected return, the optimal weight for the first risky asset is:

$$w_1 = \frac{1}{\lambda\sigma_1^2\left(1-R_1^2\right)}\left(e_1 - \sum_{i=2}^{n}\beta_{1i}e_i\right), \tag{9}$$

and for the second risky asset is:

$$w_2 = \frac{1}{\lambda\sigma_2^2\left(1-R_2^2\right)}\left(e_2 - \sum_{\substack{i=1 \\ i\neq 2}}^{n}\beta_{2i}e_i\right). \tag{10}$$

In general, for a *k*-th risky asset we have:

$$w_k = \frac{1}{\lambda\sigma_k^2\left(1-R_k^2\right)}\left(e_k - \sum_{\substack{i=1 \\ i\neq k}}^{n}\beta_{ki}e_i\right). \tag{11}$$

This is the optimal solution for the top administration that takes decisions on an n risky asset framework. It is interesting to notice that the numerator $e_k - \sum_{\substack{i=1 \\ i \neq k}}^{n} \beta_{ki} e_i$ can be seen as the constant of the regression of excess return of asset k on the excess returns of the other risky assets and the denominator $\sigma_k^2 \left(1 - R_k^2\right)$ is the residual variance of that regression.

To generate the equivalence between the solution of the top administration and of the decentralized decision made by portfolio managers, it is necessary to guarantee that wealth allocated by the top administration is the same as the wealth allocated by portfolio managers in each risky asset. We will now see some examples of how the equivalence problem may be solved.

2.1 Example 1. One portfolio manager for each risky asset

Suppose that we have a k-th portfolio manager that takes decisions regarding one risky asset, denominated asset k. Then the optimal weight for this portfolio manager would be:

$$w_{k,k} = \frac{r_k - r_{f,k}}{\lambda_k \sigma_k^2} \tag{12}$$

where λ_k is the coefficient of risk aversion of the k-th portfolio manager and $r_{f,k}$ is the risk free rate that he faces. The solution above shows that optimal weight in the risky asset is inversely proportional to the risk aversion and the level of risk and directly proportional to the risk premium offered by the risky asset.

As this portfolio manager does not take into account the overall covariance matrix there is a small probability that this portfolio will be efficient in an overall sense. A sufficient condition for efficiency would be that the optimal weight of the top administration multiplied by this wealth would be equal to the portfolio manager's optimal weight multiplied by the portfolio manager's initial endowment. This equivalence result may be expressed as:

$$\frac{r_k - r_{f,k}}{\lambda_k \sigma_k^2} W_{0,k} = \frac{1}{\lambda \sigma_k^2 \left(1 - R_k^2\right)} \left(e_k - \sum_{\substack{i \neq k \\ i=1}}^{n} \beta_{ki} e_i \right) W_0 \tag{13}$$

We can explicitly find a risk free interest rate that would make this condition true:

$$r_{f,k} = r_k - \frac{\lambda_k}{\lambda} \frac{1}{\left(1 - R_k^2\right)} \left(e_k - \sum_{\substack{i=1 \\ i \neq k}}^{n} \beta_{ki} e_i \right) \frac{W_0}{W_{0,k}} . \tag{14}$$

We could solve the problem using the portfolio manager's initial endowment instead:

$$W_{0,k} = \frac{\lambda_k}{\lambda} \frac{1}{\left(1 - R_k^2\right)} \left(1 - \sum_{\substack{i=1 \\ i \neq k}}^{n} \beta_{ki} \frac{e_i}{e_k} \right) W_0 , \tag{15}$$

where we used $r_{f,k} = r_f$ (in this case we can allow the risk free interest rate to be the same for both the top administration and the portfolio manager). Equation (14) and (15) must hold for all n portfolio managers for each risky asset. This implies that the risk free interest rate or initial endowment may be different for portfolio managers.

However, in general portfolio managers do not trade on one asset but in an asset class where there may be many assets. This motivates generalizations of the results obtained above.

2.2 Example 2. One portfolio manager trading on two risky assets

We can assume a k-th portfolio manager that takes decisions regarding two risky, assets 1 and 2. Thus the optimal weights for this portfolio manager would be:

$$w_{1,k} = \frac{1}{\lambda_k \sigma_1^2 \left(1 - R_{1,k}^2\right)} \left(e_1 - \beta_{12} e_2\right), \tag{16}$$

$$w_{2,k} = \frac{1}{\lambda_k \sigma_2^2 \left(1 - R_{2,k}^2\right)} \left(e_2 - \beta_{21} e_1\right), \tag{17}$$

where $w_{1,k}$ and $w_{2,k}$ are the optimal weights for assets 1 and 2, respectively and $R_{i,k}^2$ is the *r*-squared of the regression of excess return of the *i*-th risky asset on excess return of the other risky assets managed by the portfolio manager *k*.

Equivalence of results can be derived using the condition that the amount invested in each risky asset is the same. This condition for the first and second risky assets can be written, respectively, as:

$$\frac{1}{\lambda_k \sigma_1^2\left(1-R_{1,k}^2\right)}\left(e_1-\beta_{12}e_2\right)W_{0,k}^1=\frac{1}{\lambda\sigma_1^2\left(1-R_1^2\right)}\left(e_1-\sum_{i=2}^{n}\beta_{1i}e_i\right)W_0 \tag{18}$$

$$\frac{1}{\lambda_k \sigma_2^2\left(1-R_{2,k}^2\right)}\left(e_2-\beta_{21}e_1\right)W_{0,k}^2=\frac{1}{\lambda\sigma_2^2\left(1-R_2^2\right)}\left(e_2-\sum_{\substack{i=1\\ i\neq 2}}^{n}\beta_{2i}e_i\right)W_0 \tag{19}$$

where $W_{0,k}^1$ and $W_{0,k}^2$ are the available initial endowments that portfolio manager *k* has to invest in assets 1 and 2, respectively. We can find endogenously the amount of initial endowments that the top administration must dispose to portfolio manager *k* by solving (18) and (19):

$$W_{0,k}^1=\frac{\lambda_k}{\lambda}\frac{\frac{1}{\left(1-R_1^2\right)}\left(e_1-\sum_{i=2}^{n}\beta_{1i}e_i\right)}{\frac{1}{\left(1-R_{1,k}^2\right)}\left(e_1-\beta_{12}e_2\right)}W_0 \tag{20}$$

$$W_{0,k}^2=\frac{\lambda_k}{\lambda}\frac{\frac{1}{\left(1-R_2^2\right)}\left(e_2-\sum_{i=2}^{n}\beta_{2i}e_i\right)}{\frac{1}{\left(1-R_{2,k}^2\right)}\left(e_2-\beta_{21}e_1\right)}W_0 \tag{21}$$

It is important to notice that this initial endowment depends of the risk aversion of individual portfolio managers and the top administration.

It is interesting to notice that initial endowment available to the portfolio manager rises as the portfolio managers are more risk averse relatively to the top administration. As the ratio λ_k/λ increases, more initial endowment can be disposed to portfolio manager *k*. In general, the initial endowment disposed for portfolio manager *k* for a risky asset *m* should be given by:

$$W_{0,k}^{m}=\frac{\lambda_k}{\lambda}\frac{\frac{1}{\left(1-R_m^2\right)}\left(e_m-\sum_{\substack{i=1\\i\neq m}}^{n}\beta_{mi}e_i\right)}{\frac{1}{\left(1-R_{k,m}^2\right)}\left(e_m-\beta_{m2}e_2\right)}W_0 \tag{22}$$

2.3 Example 3. Two portfolio managers trading in the same risky asset m

An interesting example to analyze would be the case where there are two portfolio managers (*k* and *j*) trading on the same asset, say asset *m*. In this particular case, equivalence could be obtained by:

$$w_{m,k}W_{0,k}^{m}+w_{m,j}W_{0,j}^{m}=w_mW_0\,. \tag{23}$$

As there is no particular reason for the top administration to use different initial endowments for portfolio managers, we have that $W_{0,k}^{m}=W_{0,j}^{m}$ and (23) can be rewritten as:

$$W_{0,k}^{m}=W_{0,j}^{m}=\frac{\left(\frac{1}{\lambda\left(1-R_m^2\right)}\left(e_m-\sum_{\substack{i=1\\i\neq m}}^{n}\beta_{mi}e_i\right)\right)}{\left(\frac{e_m}{\lambda_k}+\frac{e_m}{\lambda_j}\right)}W_0\,. \tag{24}$$

Expression (24) could be easily generalized for any number of portfolio managers trading on any specific risky asset. The top administration may choose to give different initial endowments for different portfolio managers to invest in the same asset, in this case (23) can be rewritten as:

$$W_{0,k}^{m}=\frac{w_m}{w_{m,k}}W_0-\frac{w_{m,j}}{w_{m,k}}W_{0,j}^{m}\ . \tag{25}$$

In this case the initial endowment available for portfolio manager k would depend on the initial endowment available for portfolio manager j. We will analyze the general case in the next subsection section.

2.4 A more general case: there are l portfolio managers and n risky assets

Suppose that we have l portfolio managers and n risky assets. Portfolio managers are specialized in subsets $m_1, m_2, \ldots, m_l$ where m_k corresponds to the subset that the k-th portfolio manager trades. It is assumed that $m_k \cap m_j \neq \varnothing$ for some k, j; $k \neq j$. The restrictions that should apply are given by:

$$\sum_{k=1}^{l} w_{1,k} W_{0,k}^{1} = w_1 W_0$$
$$\sum_{k=1}^{l} w_{2,k} W_{0,k}^{2} = w_2 W_0$$
$$\vdots$$
$$\sum_{k=1}^{l} w_{n,k} W_{0,k}^{n} = w_n W_0$$

In this case the top administration decides which assets each portfolio manager should trade. The examples above help to understand how the top administration would solve his problem. If there are more than one portfolio manager in a single asset then the top administration could solve the problem as shown in example 3. If there is only one portfolio manager in a single asset then he could solve the equivalence result as shown in example 1. Finally, if portfolio managers trade in more than one asset, all that the top administration has to do is to use different initial endowment for each risky asset.

3. Interpreting the results

In this section, we do some comparative static and interpret the expressions previously found. It would be interesting to understand what happens to portfolio managers' initial endowment (or the risk free interest rate) when the exogenous parameters change.

If we use expression (15) then the initial endowment would depend on changes in the expected return for asset k as given below:

$$\frac{\partial W_{0,k}}{\partial r_k} = \frac{\lambda_k}{\lambda} \frac{1}{\left(1 - R_k^2\right)} W_0 \sum_{\substack{i=1 \\ i \neq k}}^{n} \beta_{ki} \frac{e_i}{e_k^2} . \tag{26}$$

The sign of this derivative is positive, reflecting the growth of interest in investing in risky asset k by the top administration (which can be seen from (11)). We could also answer how the wealth would change for a given portfolio manager that trades on asset k when the expected return on asset j changes:

$$\frac{\partial W_{0,k}}{\partial r_j} = -\frac{\lambda_k}{\lambda} \frac{1}{\left(1 - R_k^2\right)} W_0 \beta_{kj} \frac{e_j}{e_k} \tag{27}$$

The sign of this derivative depends on the beta coefficient, i.e., in the correlation between assets. If the correlation is negative then the top administration would increase the initial endowment to induce an increase in investment in asset k. The increase investment in an asset negatively correlated with the asset that improved its expected return is due to a hedge effect. On the other hand, if correlation were positive then the initial endowment would be reduced to reduce the risk exposure of the overall portfolio.

We can interpret expression (22), which gives the optimal initial endowment that should be available for a portfolio manager in order to obtain the equivalence result. The initial endowment available depends on the ratio of the risk aversion coefficients. The greater λ_k the more risk averse is portfolio manager k. If this portfolio manager is more risk averse than the top administration then he would be propense to underinvest in asset m. In that case the top administration would increase the portfolio manager's initial endowment in order to induce an increase in the portfolio manager's risk exposure.

$W_{0,k}^m$ also depends on the ratio of the Residual Sharpe Ratios. The greater the Residual Sharpe Ratio of the top administration relative to the Residual Sharpe Ratio of the portfolio manager for asset m the more willing the top administration is to increase the portfolio manager's initial endowment in asset m. Increasing the initial endowment would make the portfolio manager invest more in that particular asset.

From expression (29) we can derive an interesting relation between the initial endowments of portfolio managers. The partial derivative of portfolio manager's *k* initial endowment for asset *m* with respect to the initial endowment of portfolio manager *j* for asset *m* is:

$$\frac{\partial W_{0,k}^{m}}{\partial W_{0,j}^{m}} = -\frac{w_{m,j}}{w_{m,k}} = -\frac{\lambda_j}{\lambda_k}\frac{\frac{1}{\left(1-R_{m,k}^{2}\right)}\left(e_m - \sum_{\substack{i=1\\ i\neq m}}^{m_k}\beta_{mi}e_i\right)}{\frac{1}{\left(1-R_{m,j}^{2}\right)}\left(e_m - \sum_{\substack{i=1\\ i\neq m}}^{m_j}\beta_{mi}e_i\right)} \tag{28}$$

This expression gives us the rate at which the top administration would decrease (increase) portfolio manager *k* initial endowment after increasing (decreasing) portfolio manager *j* initial endowment. This trading rate is dependent on the risk aversion coefficients ratio and the Residual Sharpe Ratio Coefficients.

4. Conclusions

In general, decentralization of portfolio allocation would not generate an efficient global portfolio as decentralized decisions do no take into account the overall covariance matrix.

It is possible to use the risk free interest rate and the available initial endowment for portfolio managers in order to generate an equivalence of portfolio allocation results and find an efficient global portfolio. If portfolio managers trade in more than one risky asset then the top administration could use the initial endowment available for investing in each risky asset as a control variable to obtain the equivalence result. This means that the top administration could redistribute the initial endowment among portfolio managers as exogenous parameters change at the beginning of the portfolio building process. We used here a type of second welfare theorem.

Our findings suggest that the risk free interest rate and the initial endowment used as control variables depend on a number of parameters and on risk aversion coefficients. This motivates further research on estimation of risk aversion coefficients. As it is widely known these coefficients are not directly observable and to our knowledge there are not many published work that estimates individual risk aversion coefficients[4]. This is left for further research.

Another interesting question would be to answer what is the optimal number of portfolio managers that a firm should have. This is a very important problem that trading firms deal with all the time and is still an open question.

Agency considerations and the use of multi-period portfolio selection models would be another important route to explore within decentralized investment management[5]. However, our approach could be use in a dynamic framework. The top administration must define the investment horizon for the firm and other portfolio managers and at the end of each period he would coordinate and redistribute initial endowment among portfolio managers. Nonetheless, this extension raises the question of how well portfolio managers would have performed within the investment horizon period and this would lead to asymmetric information considerations among portfolio managers as well.

There are many open questions yet. As the literature on this theme is almost nonexistent, we answered very simple questions. This is a first step, yet important, towards an understanding of how delegation of portfolios can be made without loosing overall efficiency. However, there are many questions to be made and answered. They are left for further research.

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4 Sharpe et alli (1999) derive the risk tolerance for an investor using the equation for an indifference curve of an investor having constant risk tolerance. Their solution depends of the optimal weights given by managers for risky assets, on the variance and expected excess return of the portfolio.

5 Sharpe (1981) analyzes decentralized investment management in a different framework. Konno and Yamazaki (1992), Porter (1973), Pyle and Turnovsky (1970), Roy (1952) and Pye (1972) analyze different approaches to the mean-variance portfolio criteria such as the safety-first and stochastic dominance criteria.

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