ISSN 1518-3548

BANCO CENTRAL DO BRASIL

Working Paper Series

3

Private Sector Participation: a Theoretical Justification of the Brazilian Position

Sérgio Ribeiro da Costa Werlang

July, 2000

ISSN 1518-3548

CGC 00.038.166/0001-05

Working Papers Series	Brasília	n. 3	Sep	2000	P. 1 – 19

Working Paper Series

Edited by:

Research Department (Depep)

(e-mail: conep.depep@bcb.gov.br)

Reproduction permitted only if source is stated as follows: Working Paper Series n. 3.

General Control of Subscription:

Banco Central do Brasil
Demap/Disud/Subip
SBS – Quadra 3 – Bloco B – Edifício-Sede – 2º subsolo
70074-900 - Brasília (DF)
Telefone: (61) 414-1392
Fax: (61) 414-3165

Number printed: 450 copies

The views expressed in this work are those of the authors and do not reflect those of the Banco Central or its members.

Although these Working Papers often represent preliminary work, citation of source is required when used or reproduced.

Banco Central do Brasil Information Bureau

Address:	Secre/Surel/Dinfo
	Edifício-Sede, 2º subsolo
	SBS - Quadra 3, Zona Central
	70074-900 - Brasília (DF)
Phones:	(61) 414 (....) 2401, 2402, 2403, 2404, 2405, 2406
DDG:	0800 992345
FAX:	(61) 321 9453
Internet:	http://www.bcb.gov.br
E-mail:	cap.secre@bcb.gov.br
	dinfo.secre@bcb.gov.br

Private Sector Participation: A Theoretical Justification of the Brazilian Position

By Sérgio Ribeiro da Costa Werlang*

Abstract

The Brazilian position on the issue of private sector participation in the efforts to forestall and resolve emerging markets crises is that there should be an approach of contacting and convincing a large number, but not the totality, of private creditors. They should be convinced that the international public sector loans will allow a transition to stability that is overfinanced in case they voluntarily join in by maintaining their exposure to the country, at the spreads they choose, and with the counterparts they wish to have as clients. We model private sector participation by means of a game. We show that the traditional argument that there is a coordination problem among the private creditors does not exist in a model without Knightian uncertainty, because there is a unique Pareto dominant Nash equilibrium that involves participation. By introducing Knightian uncertainty, we show that if the degree of uncertainty, as measured by the uncertainty aversion, is high enough, then there is a unique Nash equilibrium under uncertainty, which involves nonparticipation. Finally, we show that if there is a large enough number of private creditors who decrease their uncertainty aversion, then again private participation becomes the unique Pareto dominant Nash equilibrium under Knightian uncertainty. If we interpret the approach of contacting and convincing the private creditors as decreasing their uncertainty aversion, then this last result is a justification of the Brazilian position. In fact, the private creditors would voluntarily choose to maintain their exposures, because private sector participation is the unique Pareto dominant Nash equilibrium under uncertainty of the game.

* Banco Central do Brasil, EPGE, Fundação Getúlio Vargas.

Private Sector Participation: A Theoretical Justification of the Brazilian Position

1. The Brazilian Position

This paper draws upon the previous works on private sector participation presented at the Bonn meeting of the extended G-22, on March 11. In particular, we will be referring to the contributions of the Canadian delegation (Canada (1999)), the French delegation (France (1999)) and the IMF (IMF (1999a, 1999b)).

The Brazilian position on the issue of private sector participation in the efforts to forestall and resolve emerging markets crises is that there should be an approach of contacting and convincing a large number, but not the totality, of private creditors. They should be presented with the agreements between the country and the international financial institutions (IFIs), and shown that the international public sector loans will allow a transition to stability that is overfinanced in case they voluntarily join in. Joining in, in this situation, means that they are supposed to maintain their exposure to the country, at the spreads they choose, and with the counterparts they wish to have as clients. The Brazilian position is that such a choice should be voluntary, and that they will be willing to join by the sheer force of the argument that they will make more money if they do so.

We recognize that the cost of taking measures that require involuntary action is very high, from past Brazilian experience. In fact, we have tried all possible heterodoxies in the eighties and the early nineties. We have had Malaysia-like capital controls in 1983, and the result was that for the eleven years following (1984 -1994) the average current account deficit was close to zero. In addition, we have had an external moratorium in 1987, which also did not help our creditworthiness, and did not solve our fiscal problems (as was propounded by those who implemented it). The result of this and other nonmarket-oriented measures was always for the worse. Nonmarket, involuntary measures not only are extremely costly in terms of economic efficiency (many broken contracts), but also in the end they are not able to stabilize the economy.

As a matter of fact, both IMF documents (IMF (1999a and 1999b)) explicitly recognize that the mere possibility that involuntary measures could have been used in the case of Brazil, may have contributed to the relatively low rollover rates in October 1998, before the announcement of the details of the Fund supported adjustment program. Moreover, notice that this happened despite the fact that the Brazilian authorities repudiated up front the use of nonmarket measures.

Although the aversion for nonmarket-oriented measures is generally well accepted, in regards to private sector participation the story is different. The Brazilian position is far from being devoid of controversy. For example, Canada (1999) specifically emphasized the need for a framework that would permit countries experiencing a massive capital outflow to declare a temporary standstill on debt repayments. France (1999) distinguishes the situations, creating a typology of crises. This is a promising path, and both the Canadian delegation and the IMF documents have implicit in their arguments a classification of cases, according to the intensity of the crisis. However, in the conclusion of the French delegation's paper, they show concern that market-friendly instruments may not be enough to solve the problem, even in some of the "benign" instances.

We model private sector participation by means of a game. The players of the game are the private creditors. We show that the traditional argument that there is a coordination problem among the private creditors does not exist in a model without Knightian uncertainty, because there is a unique Pareto dominant Nash equilibrium that involves participation. By introducing Knightian uncertainty, we show that if the degree of uncertainty, as measured by the uncertainty aversion, is high enough, then there is a unique Nash equilibrium under uncertainty, which involves nonparticipation. Finally, we show that if there is a large enough number of private creditors who decrease their uncertainty aversion, then again private participation of these banks becomes the unique Pareto dominant Nash equilibrium under Knightian uncertainty.

If we interpret that the approach of contacting and convincing the private creditors as a means of lowering their uncertainty aversion (or their degree of uncertainty), then this last result may be interpreted as a justification of the Brazilian position. In fact, the private creditors would voluntarily choose to maintain their exposures, because private sector participation is the unique Pareto dominant Nash equilibrium under uncertainty of the game. Therefore, the model shows that the Brazilian position makes sense in many cases, i. e., market-friendly, voluntary participation is possible to be obtained in many situations.

Again, the model also allows one to conclude that there may be situations of a country in such extreme financing problems that only a nonmarket forced measure would do the job, because it could be the case that it would be impossible to lower the uncertainty aversion enough to generate the good Pareto dominant Nash equilibrium. Hence, in extreme situations the model predicts that only involuntary measures, like the concerted rollover that was employed in the case of Korea, can do the job. This means that the model agrees with the intuition, so that it is a suitable tool to analyze the problem of private sector participation.

The paper is divided as follows. The next section lays out the model, and solves it without Knightian uncertainty. Here we show that the usual view of a coordination problem is not present in traditional models without Knightian uncertainty. Then, section three introduces Knightian uncertainty, and the two main results are derived. The appendix contains some material on Knightian uncertainty, enough to guide the reader through the main text. Finally, section four concludes.

2. The Model

As Canada (1999) points out, two economic problems may justify private sector participation. The first is a long run problem, that of moral hazard - if a creditor knows that there is going to be bailing out, and then there is no incentive to be careful with the loans. The same problem arises in modern banking systems. It turns out that in the case of Brazil this is not the central problem. The reason is simple: this is a structural long run problem, and the effects on a country would be visible only if the size of the bank lines had increased substantially, at a lower cost than before. It turns out that trade related lines in Brazil remained approximately the same from 1995 until the Russian crisis, with no substantial change in the spread. Thus, if we are interested in the economic problem of private sector participation in Brazil, this is not the most relevant aspect to consider.

The second is the tendency of markets to display "herding" behavior, including self-fulfilling "creditor runs". We want to model creditor runs, clearly an important phenomenon in the case at hand, because we observed an unusual decrease of trade related lines in Brazil after the Russian crisis.

The model is as follows. After the adjustment the country is such that the balance of payments is fully financed in the short and long run by direct investments and the loans of the international financial institutions, if the private creditors (i. e., banks) maintain their exposure to the country in trade related lines. In fact, let us assume that there is a yearly financing surplus of Q. That is to say, the banks may decrease their aggregate position still by Q and the balance of payments in the short run would be still financed. Q is the amount of overfinancing in case the banks join in the effort. Suppose there are n identical banks, each one of them with a trade related exposure of b to the country. The players of the game are the banks. To simplify the analysis, we assume that any bank has only two alternatives: either keep the exposure, or reduce it to zero. Notice that we are not considering some intermediate choices like: (a) reducing the exposure but not to zero; (b) increasing the spreads; (c) reducing the average maturity of the loans; and (d) the bank cannot leave and come back, if it reduces the exposure to zero, it does so forever. Obviously, all this aspects could be considered in models that are more involved.

The payoff to bank i, where i is a number between 1 and n, is defined as: (i) if bank i reduces the exposure to zero, the payoff is 0;
(iia) if the bank keeps the exposure, and the number of other banks who decide to reduce their exposure is less than or equal to the greatest integer less than or equal to Q/b (which is also known as the integer part of Q/b, which we denote as [Q/b]), then bank i gets s.b/r, where s is the spread related to the transaction, and r is the international interest rate (remember - if the bank does not keep the exposure, it reduces to zero for

good, so that it loses the present value of the flow of spreads, which means that the gain from keeping is also the present value);

(iib) if the bank keeps the exposure, and the number of other banks who decide to reduce their exposure is greater than [Q/b], then bank i loses the loan, because the country is unable to fulfill its foreign currency obligations, which means the payoff is -b.

Observe that we are considering, again for sake of simplicity, that the country does not pay any of the banks in the case there is not enough of them to maintain the exposure. It is an obvious exaggeration, because, for example, the country could choose to pay them proportionately to their exposures. However, for the purpose of the analysis, this will suffice. The solution of the game in the absence of Knightian uncertainty is given by the proposition below.

Proposition 1. If $0 \leq [Q/b] \leq n-1$, then there are only two Nash equilibria for this game. First, all of the banks keep their exposure, in which case each of them gets s.b/r. Second, all of the banks decide to reduce their exposure, in which case each of them gets 0. Therefore, there is a unique Pareto dominant Nash equilibrium of the game in which banks keep their exposure.

Proof. First, we show that in equilibrium it cannot occur that some banks stay, some others leave. In fact, assume that there is an equilibrium in which there are two banks, one which keeps and the other which reduces their lines. If the bank that keeps is maximizing its payoff, then the number of other banks who reduce their exposure is less than or equal to [Q/b]. Thus, consider the case of the bank that does not keep the lines. In the perspective of this bank, the number of other banks who do not keep the lines has to be less than or equal to [Q/b]-1, which is itself a number less than or equal to [Q/b]. Hence, by not keeping this bank is obtaining 0, while if this bank kept the exposure, it would get s.b/r, which is a larger number. This would mean that this bank would not be optimizing, which is a contradiction. Therefore, there cannot be an equilibrium where the banks are acting differently. Now one has to show that both situations, where all of them keep their exposure, and where all of them reduce their exposure are Nash equilibria. But this follows immediately from the definition of the payoffs, and from the assumption that $0 \leq [Q/b] \leq n-1$. QED.

This proposition allows us to claim that the traditional coordination problem does not arise. In fact, it suffices that the banks know the structure of the game to see that they have only two Nash equilibria, and that there is one in which all of them make more money. There is no reason to suppose they would choose the worst equilibrium. The problem in the mind of everyone, when one speaks of possible coordination failure in a case as such, is different. People have in mind something akin to Knightian uncertainty, and that is the topic of the next section.

3. Introducing Knightian Uncertainty

Since the beginning of the nineties Knightian uncertainty started being used to analyze economic phenomena. The appendix describes the main results. Uncertainty, as originally defined by Frank Knight (Knight (1921)), is a situation where agents decide without knowing a probability distribution of unknown factors. As opposed to it, Knight defines decision under risk as the case in which agents decide with the knowledge of a probability distribution behind the unknown factors. The effect of uncertainty in economic models has being ignored, mainly because of the very influential book of Leonard Savage (Savage (1954)). There, Savage shows that under certain conditions, decision under uncertainty (Knightian uncertainty, as we will call it) reduces to decision under risk, where the risk is subjective. More recently, since the works of David Schmeidler and Itzhak Gilboa, a richer model of Knightian uncertainty was introduced in the economic literature. In the econometric literature, Knightian uncertainty is equivalent to a generalized version of robustness analysis.

The analysis of games under uncertainty (we will use freely the terms uncertainty and Knightian uncertainty as having the same meaning) may be summarized as saying that the players get more cautious. Extreme caution is translated into the decision-theoretic maxmin behavior. That is to say, an agent has extreme caution if he acts to maximize his utility, but taking into account that the worst possible outcome of his actions. In other words, for any action a taken, the agent considers the combination of factors that yields him the lowest possible utility given that action a. Then he chooses the a that maximizes the utility. This is why the name is maxmin: it maximizes the minimum possible utility.

In the game above, there is a very interesting phenomenon: the extremely cautious (maxmin) behavior of every bank is the same - reduce to zero its exposure. Thus, there is a Nash equilibrium that turns out to be also the maxmin behavior. In a simplified version of the uncertainty model of Schmeidler-Gilboa, the behavior of the players is a weighted average between the usual behavior and maxmin behavior. That is, the players behave as though they had a weight $0 \leq c_i \leq 1$ such that the "new" payoff is $(1-c_i)$ times the old payoff plus c_i times the worst that can happen in case the action is chosen. In the appendix, we derive this result, and many more details are given about Schmeidler-Gilboa's theory of Knightian uncertainty.

The parameter c_i is known as the uncertainty aversion of player i (as defined by Dow and Werlang (1992a)). The closer this parameter is to zero, the more the players behave as if there were no uncertainty. On the other hand, the closer c_i is to 1, the more cautious player i is, that is to say, the more averse to uncertainty he is. We may also think that the parameter c_i measures the degree of uncertainty of player i.

This alters the game, and the payoff to bank i, is now modified to:
(i) if bank i reduces the exposure to zero, the payoff is 0, because this coincides with the worst that can happen to it in the case of not keeping the exposure;
(iia) if the bank keeps the exposure, and the number of other banks who decide to reduce their exposure is less than or equal to [Q/b], then bank i gets $(1-c_i).s.b/r + c_i .(-b)$, because the worst that can happen in this case is that more than [Q/b] banks will pull out of the country;
(iib) if the bank keeps the exposure, and the number of other banks who decide to reduce their exposure is greater than [Q/b], then bank i's payoff is -b, because this is already the worst that can happen to it under these circumstances.

The solution of this game is given below.

Proposition 2. Let k be the number of i's such that the uncertainty aversion parameter c_i is less than s/(s+r). Suppose that there are n-k of the banks such that c_i is greater than s/(s+r). We avoid limit cases of equality, because they are obvious, but tedious to deal with.

(i) If k is less than n - [Q/b], then there is only one Nash equilibrium under uncertainty, where all banks reduce their exposure to zero.

(ii) If k is larger than or equal to n - [Q/b], then there are only two Nash equilibria under uncertainty: one where all banks reduce to zero their exposure, and the other where all k banks with low uncertainty aversion keep their exposure, but the other n-k do not. The latter equilibrium is the Pareto dominant Nash equilibrium under uncertainty.

Proof. If the bank is such that its uncertainty aversion parameter is greater than s/(s+r), then it is easy to see that reducing to zero the exposure is a strictly dominant strategy. This takes care of the case where k is less than n - [Q/b], because n-k will be greater than [Q/b], which means that there will be more than [Q/b] banks that will reduce to zero the exposure, so that even the banks with low uncertainty aversion will have as an optimizing choice the reduction of the exposure to zero. This shows (i). To show (ii), notice that for the banks with low aversion to uncertainty (i. e., $c_i < s/(s+r)$), the optimal decision is the same as the banks with zero uncertainty aversion. Hence, the result is easy to see.

QED.

This proposition allows us to analyze private sector participation in a more realistic setup. First, there is a cutoff level of the uncertainty aversion, s/(s+r), such that if the banks have a higher uncertainty aversion, they will pull out of the country, which means that if there are enough of those, the other banks will find optimal to reduce their exposure to zero too. This cutoff level is increasing in the spread, and decreasing with the international interest rates. Note that for the values of 2% per year for the spread, and 5.5% per year for the interest rate (approximate dollar values), we have the cutoff of the uncertainty aversion parameter at 0.267, a relatively high number. The uncertainty aversion of the banks may be interpreted as reflecting their fear that there will be problems with the implementation of the international financial institutions' program. Here we also see that allowing the banks to set the spreads freely may be a powerful incentive to increase the cutoff of the uncertainty aversion parameter, which means a better chance of obtaining the good equilibrium.

Second, if enough of the banks (at least n - [Q/b]) have an uncertainty aversion which is smaller than the cutoff level, then we again get the good case, in which there will be a

Pareto dominant equilibrium with private sector voluntary participation. It is here that we interpret this result as a justification of the Brazilian viewpoint. The Brazilian position may be seen as a device to decrease the uncertainty aversion parameter of the banks, by means of a very close contact, where all the details of the international financial help are shown. Obviously, the game assumes implicitly that there is full monitoring of the actions of others. This is also an interpretation of the role of monitoring systems, to have reality resemble the game the most possible. Additionally, the banks will be more likely to participate the more slack they see in the program, so that short term delays in the decision to maintain the lines would not substantially affect the country.

All of the conditions above are met in the case of Brazil. In addition, it is fundamental, as time goes by, that the country fulfills all its commitments to the international financial institutions.

4. Conclusion

We have shown a game theoretic model that we believe to be the appropriate framework to study private sector participation. From the game theoretic viewpoint, two are the facts we implicitly assume. First, in a coordination game with two Nash equilibria, where one is a Pareto dominant equilibrium, then this will be the equilibrium chosen by all the participants (note that this used to be a big controversy among game theorists during the eighties, but this discussion is considered somewhat sterile right now). Second, the introduction of Knightian uncertainty captures all the intuition of those who thought that the dominated Nash equilibria without uncertainty could be chosen. In this case the Pareto dominated Nash equilibrium is also the maxmin (extremely cautious) equilibrium. This is the real dilemma in people's mind: caution X profit. Knightian uncertainty allows a precise modeling of this phenomenon.

The players of the game are the private creditors, or banks. We show that the traditional argument that there is a coordination problem among the private creditors does not exist is in a model without Knightian uncertainty, because there is a unique Pareto dominant Nash equilibrium that involves participation. By introducing Knightian uncertainty, we show that if the degree of uncertainty, as measured by the uncertainty aversion, is high enough, then there is a unique Nash equilibrium under uncertainty, which involves nonparticipation. We also show that if there is a large enough number of private creditors who decrease their uncertainty aversion, then again private participation of these banks becomes the unique Pareto dominant Nash equilibrium under Knightian uncertainty.

Finally, the uncertainty aversion parameter is a perfect one-dimensional tool to classify the countries according to the severity of the crisis it faces. Given that, the intuition behind the works of Canada (1999), France (1999) and IMF (1999a and 1999b) may be translated into this model. The model also allows one to clarify the role of monitoring: to disseminate the knowledge about the actions of the other players of the game. And the fact that in many cases the voluntary approach to the problem of private sector participation may be successful is also shown not to be incompatible with the intuition of the works already cited. Hence, this is a theoretical justification of the Brazilian position.

Appendix: Nash Equilibrium under Knightian Uncertainty

This appendix draws upon material from Dow and Werlang (1992b), Dow, Simonsen and Werlang (1993) and Dow and Werlang (1994). Schmeidler (1982, 1989) and Gilboa (1987) have developed an axiomatic model of rational decision-making in which agents' behavior distinguishes between situations where agents know the probability distributions of random variables and situations where they do not have this information. We refer to the former as risk and the latter as uncertainty, or Knightian uncertainty (as defined by Knight (1921)). Synonyms that are used in the literature include roulette lottery, for risk, and horse lottery and ambiguity, for uncertainty. The traditional model of uncertainty used in economics is that of Savage (1954), which reduces all problems of uncertainty to risk under a subjective probability. The axiomatization of Schmeidler-Gilboa leads to very distinct behavior: behavior under uncertainty is inherently different from behavior under risk. We now give a brief exposition of the main aspects of their model. The reader is referred to the papers by Schmeidler and Gilboa cited above for a complete description and for the underlying axioms, and to Dow and Werlang (1992a), which contains an example and an application to portfolio choice (it also includes a mathematical appendix with all basic material on non-additive probabilities). Dow and Werlang (1992b) have an explanation of the excess volatility puzzle, and Simonsen and Werlang(1991) also describe the implications for portfolio choice.

The Schmeidler-Gilboa model predicts that agents' behavior will be represented by a utility function and a (subjective) non-additive probability distribution. A non-additive probability P reflecting aversion to uncertainty satisfies the condition

$$P(A) + P(B) \leq P(A \cup B) + P(A \cap B), \qquad (*)$$

rather than the stronger condition satisfied by (additive) probabilities

$$P(A) + P(B) = P(A \cup B) + P(A \cap B).$$

In particular, $P(A) + P(A^c)$ may be less than 1; the difference can be thought of as a measure of the uncertainty aversion attached by the agent to the event A. The *uncertainty aversion of P at event A* is $c(P,A) = 1 - P(A) - P(A^c)$ (Dow and Werlang(1992a)).

All the non-additive probabilities considered in this paper will reflect uncertainty aversion, i.e. they will satisfy inequality (*). In addition, we will restrict attention to the case of a finite set of states of the world.

The agent maximizes expected utility under a non-additive distribution, where the expectation of a non-negative random variable X is defined as:

$$E[X] = \int_{\Re+} P(X \geq x) dx .$$

Associated with a non-additive probability P is a set Δ of additive probabilities called the core of P, which is defined (analogously to the core in cooperative game theory) as the set of additive probability measures π such that $\pi(A) \geq P(A)$ for all events A. If the non-additive probability satisfies the inequality (*) (reflecting aversion to uncertainty) the core is non-empty. A closely related model of behavior under uncertainty is for the agent to act to maximize the minimum value, over the elements of the core, of expected utility (Gilboa and Schmeidler (1989)).

The support of a non-additive probability P may be defined analogously to the additive case.

Definition: a *support* of a non-additive probability P is an event A such that $P(A^c) = 0$ and $P(B^c) > 0$ for all events $B \neq A$ with $A \supset B$.

It should be clear that there might be several supports. Also note that any support must be contained in the smallest set S such that $P(S)=1$.

Now, let us move to games. The concept of Nash equilibrium under uncertainty was proposed by Dow and Werlang (1994). Broadly speaking, it is a weighted average of Nash and maxmin that unifies two apparently conflicting views of rational behavior in a game. Let us develop the heuristics of the idea, and then proceed to a formal definition.

Let $\Gamma=(A_1, A_2, u_1, u_2)$ be a two-person finite game (also known as a bi-matrix game) where the A_i's are pure strategy sets and u_i's are utilities (payoffs). This will be called the primitive game, or game without uncertainty. For i=1 and 2, define $\underline{u}_i(a_i) = \min_{a_j \in A_j} u_i(a_i, a_j)$, being $i \neq j$. This amount is what player i will get in case she plays strategy a_i against "the devil". ("The devil" is a fictitious player whose objective is to always choose the action that will hurt player 1 the most.) The game with constant degree of uncertainty (measured by the uncertainty aversion) c_1 for player I and c_2 for player II ($0 \leq c_1 \leq 1$ and $0 \leq c_2 \leq 1$) is the bi-matrix game $\Gamma'=(A_1, A_2, u'_1, u'_2)$, where $u'_i(a_i, a_j) = (1-c_i) \cdot u_i(a_i, a_j) + c_i \cdot \underline{u}_i(a_i)$, $j \neq i$, i=1 and 2.

A Nash equilibrium in this new game is defined as a Nash equilibrium under uncertainty in the primitive game. An interpretation of this new game is that each player attributes a certain probability that the other player will behave irrationally, acting like the devil. With zero uncertainty one gets the usual definition of Nash equilibrium. With 100% uncertainty for every player, an equilibrium is a combination of maxmin strategies.

It is easy to check that if $(\tilde{a}_1, \tilde{a}_2)$ is at the same time a Nash equilibrium and a combination of maxmin strategies of the primitive game, then it is also a Nash equilibrium of the game with any given constant degree of uncertainty.

Let us now generalize the definition of Nash equilibrium under uncertainty. The point of departure will be a well-known definition of mixed strategy in standard theory: an additive probability on the space of pure strategies of the player. As before, we restrict attention to two-person finite games $\Gamma=(A_1, A_2, u_1, u_2)$. In the standard theory, a mixed strategy Nash equilibrium can be defined as follows. Let (μ_1, μ_2) be a pair of (additive) probability measures and let supp[μ_i] denote the support of μ_i. In Nash equilibrium, every $a_1 \in$ supp[μ_1] is a best response to μ_2, i.e. a_1 maximizes the expected utility of player 1 given that player 2 is playing the mixed strategy μ_2; conversely, every $a_2 \in$ supp[μ_2] is a best response to μ_1. A subjective interpretation can be given to the Nash equilibrium: the mixed strategy of player 1, μ_1, may be viewed as the beliefs that player 2 has about the pure strategy play of player 1. Conversely, the mixed strategy of player 2, μ_2, may be viewed as the beliefs player 1 has about the pure strategy play of player 2.

Now, under uncertainty, what happens is that each player no longer views the strategy of the other player as an additive, but as a subadditive probability on the other player's strategy space. Moreover, we have assumed up to now that the degree of uncertainty c(P,A) is constant for each player. This assumption can be lifted in a general definition. The definition below appeared before in Dow and Werlang (1994).

Definition: We say that a pair (P_1, P_2) of subadditive probabilities (all our non-additive probabilities will satisfy inequality (1) above), P_1 over A_1 and P_2 over A_2 is a *Nash Equilibrium under Uncertainty* if there exists a support of P_1 and a support of P_2 such that:

(i) for all a_1 in the support of P_1, a_1 maximizes the expected utility of player 1 given that player 1 beliefs about the strategies of player 2 are P_2, and conversely;

(ii) for all a_2 in the support of P_2, a_2 maximizes the expected utility of player 2 given that player 2 beliefs about the strategies of player 1 are P_1.

The definition above reduces to the standard definition of Nash equilibrium, whenever there is no uncertainty (which means that the P's are additive). One could speculate why we have not used the smallest set of probability one instead of a support in the definition above. The reason is that this set is "too large", and the equilibrium notion thus resulting would be too strong, as well. In fact, the great strength of non-additive models is that an event may be infinitely more likely than its complement, but still have probability less than one. Take, for example, the case of a strategy set with two elements, a and b. Suppose P(a)=0.8 and P(b)=0. If we want to be "sure" that an event is going to happen, then this event has to be the whole strategy set, because it is the smallest set with probability one. However, the likelihood that the strategy a is going to be used is infinite relative to b (i.e. the relative likelihood that strategy b is going to be used is zero). Thus, in this case, it would be fair to interpret that strategy b has no chance of happening. Clearly, a standard mixed strategy Nash equilibrium is also a Nash equilibrium under uncertainty.

In addition, it is easy to see that the definition above reduces to the heuristic definition given before. In fact, consider the case of a uniform squeeze, i.e., P(A) = (1-c)Q(A), for A distinct from the whole set of strategies. From Dow and Werlang (1992a) uniform squeezes have constant uncertainty aversion equal to c and the expected value of a positive random variable X is given by

$$E_P[X] = c\cdot\min X + (1-c)\cdot E_Q[X].$$

If we find the Nash equilibria under uncertainty where the subadditive probabilities of the players are in this class, we obtain the same equilibria as above. (To see that is simple: one has to check that the (standard) mixed strategy Nash equilibria of the modified game (Γ') correspond to the Q's of the Nash equilibria under uncertainty of the primitive game.) Hence, we have the following theorem (Dow and Werlang (1994)).

Theorem. Let $\Gamma=(A_1, A_2, u_1, u_2)$ be a two-person finite game, and $(c_1, c_2) \in [0,1] X[0,1]$. Then, there exists a Nash equilibrium (P_1, P_2), where both P_1 and P_2 exhibit constant uncertainty aversion, such that c_1 is the uncertainty aversion of P_2 and c_2 is the uncertainty aversion of P_1. The reason for the interchange in the subscripts is that P_2 is what player 1 thinks player 2 is going to do, so that the uncertainty aversion of P_2 is a characteristic of player 1, and vice-versa.

Furthermore, in the heuristic discussion above, we have a practical method to compute Nash equilibria under Knightian uncertainty - just modify the game Γ to Γ', and calculate the usual mixed strategy Nash equilibria of Γ'. This is the procedure used in the text.

Bibliography

Canada (1999), "Mechanisms for Private-Sector Involvement in the Resolution and Prevention of International Financial Crises". Canadian Ministry of Finance, February 25. Presented at the Bonn seminar of the extended G-22 on March 11, on the new architecture of the international financial system.

Dow, James and Sérgio Ribeiro da Costa Werlang (1992a), "Uncertainty Aversion, Risk Aversion and the Optimal Choice of Portfolio", Econometrica 60: 197-204.

Dow, James and Sérgio R. C. Werlang (1992b), "Excess Volatility of Stock Prices and Knightian Uncertainty", European Economic Review, Proceedings of the European Economic Association Meeting in Cambridge.

Dow, James and Sérgio Ribeiro da Costa Werlang (1994), "Nash Equilibrium under Knightian Uncertainty: Breaking Down Backward Induction", Journal of Economic Theory 64: 305-324.

Dow, James, Mario Henrique Simonsen and Sérgio R. C. Werlang (1993), "Knightian Rational Expectations, Inflationary Inertia and Money Neutrality", mimeo, EPGE/FGV.

France (1999), "Implications of the Private Sector: A French Contribution". French Delegation. Presented at the Bonn seminar of the extended G-22 on March 11, on the new architecture of the international financial system.

Gilboa, Itzhak (1987), "Expected Utility with Purely Subjective Non-Additive Priors", Journal of Mathematical Economics 16: 279 - 304.

Gilboa, Itzhak and David Schmeidler (1989), "Maxmin Expected Utility with a Non-Unique Prior", Journal of Mathematical Economics 18: 141- 153.

Gilboa, Itzhak and David Schmeidler (1992), "Additive Representations of Non-additive Measures and the Choquet Integral", Northwestern University working paper.

IMF (1999a), "Involving the Private Sector in Forestalling and Resolving Financial Crises - Further Considerations". International Monetary Fund, February 25, EBS/99/21.

IMF (1999b), "Involving the Private Sector in Forestalling and Resolving Financial Crises - Further Considerations - Background Paper". International Monetary Fund, March 5, EBS/99/29.

Knight, Frank (1921), Risk, Uncertainty and Profit. Boston: Houghton.

Savage, Leonard J. (1954), The Foundations of Statistics. New York: John Wiley. (Second edition 1972, New York: Dover.)

Schmeidler, David (1982), "Subjective Probability Without Additivity" (Temporary Title), Working Paper, The Foerder Institute for Economic Research, Tel Aviv University.

Schmeidler, David (1989), "Subjective Probability and Expected Utility Without Additivity", Econometrica 57: 571 - 587.

Simonsen, Mario Henrique and Sérgio Ribeiro da Costa Werlang (1991), "Subadditive Probabilities and Portfolio Inertia", Revista de Econometria 11: 1-20.