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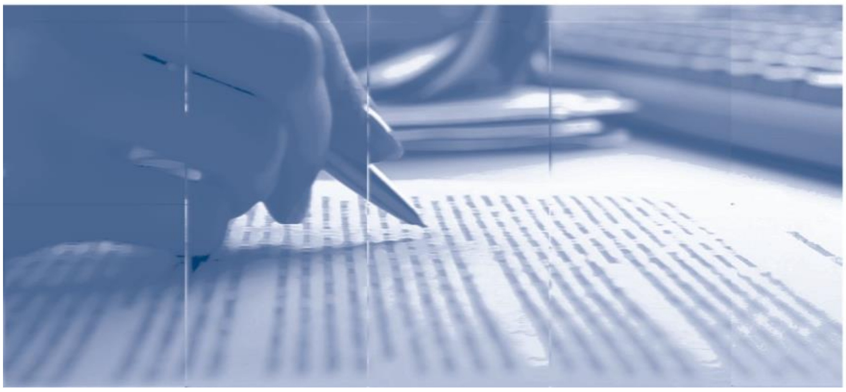

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Evaluating Systemic Risk using Bank Default Probabilities in Financial Networks

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Abstract

In this paper, we propose a novel methodology to measure systemic risk in networks composed of financial institutions. Our procedure combines the impact effects obtained from stress measures that rely on feedback centrality properties with default probabilities of institutions. We also present new heuristics for designing feasible and relevant stress-testing scenarios that can subside regulators in financial system surveillance tasks. We develop a methodology to extract banking communities and show that these communities are mostly composed of non-large banks and have a relevant effect on systemic risk. This finding renders these communities objects of interest for supervisory activities besides SIFIs and large banks. Finally, our results provide insights and guidelines that can be useful for policymaking.

Keywords: systemic risk, financial stability, interbank market, stress test, macropru-dential, network.
JEL Classification: G21, G23, C63, L14.

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1 Introduction

The occurrence of international financial crises in recent years has highlighted the need of understanding and assessing systemic risk. Besides the identification of mechanisms that may lead a financial system to a systemic crisis, it stands as an important task to identify the potential financial institutions (FIs) that may play a key role on a crisis onset. Furthermore, it is essential to have tools for assessing financial system conditions at any given time: is it next to a crisis? Is it possible to intervene to mitigate this risk and assure the financial stability? How do we optimally accomplish that, using the minimum possible resources to reach the desired effect?

Systemic crises usually begin in a single or small group of FIs and spread to a larger portion of the financial system, eventually affecting the real sector. Besides the surveillance of individual FIs, it is necessary to identify contagion mechanisms and define actions to mitigate the effects that systemic outbreaks provoke. The literature has been concerned with the possibility that the manner in which FIs relate to each other in a network is relevant to the contagion process. See, for instance, Boss et al. (2004); Furfine (2003); Inaoka et al. (2004); Soramäki et al. (2007).

Interbank markets play an essential role in a well-functioning integrated financial system through the provision of liquidity among banks. FIs lend or borrow money among themselves and make commitments of repayments at the due dates. If an FI fails in the repayment of its loans, its creditors may have trouble in honoring their debts, propagating the effects of the original failure to other institutions, in a contagion process. Problems affecting one institution may spread to other ones and even to institutions across international borders.

The contribution of this work is threefold. The first contribution is the proposal of a novel network-based scheme for evaluating systemic risk, which is inspired by the well-known DebtRank methodology (Battiston et al. (2012b)) and Merton structural model (Merton (1974)). Our framework combines financial stress levels of banks, which we evaluate using network measures that rely on feedback centrality mechanisms, together with the default probability (DPs) of banks, which we compute using banks' balance sheets. We then estimate the systemic risk using the expected impact of the financial system. We motivate the use of a network-based approach because it is able to capture topological aspects of the data relationships, which in turn may help in extracting nonlinear features of the risks embodied into the FIs relationships (Silva and Zhao (2012, 2015)). We analyze this new scheme in the Brazilian interbank market network. With this modification, we move forward with respect to the methodology presented by Battiston et al. (2012b), by modulating the impact effects estimated by their DebtRank methodology proportionally to the DPs of the institutions. To the best of our knowledge, this is the first paper that evaluates systemic risk by combining DPs and loss given default (LGD) using network analysis methodologies.

The main finding is that medium-sized banks contribute more to the systemic risk in the Brazilian interbank market network, followed by small and large banks, in this order. The reasoning behind this is that, even though large banks inflict the largest financial stress levels, their DPs are very small, yielding a very small modulated expected impact in our proposed framework. Opposed to that, medium- and small-sized banks, though only causing moderate or small financial stress levels, hold non-negligible DPs. Putting together these two indicators make their contribution to the systemic risk in the interbank market superior to that of large banks.

The second contribution of this paper is the development of new heuristics for designing stress-testing scenarios using network analysis tools. From a regulator viewpoint, a key issue in stress testing is how to develop credible and relevant scenarios that can help in assessing overall risks for the banking system. In this respect, most research to date has focused on how to calculate expected losses given that fixed initial scenario occurred. There is very little discussion in the literature on how to construct scenarios, which may be relevant for stress-testing purposes. A relevant scenario for banking stress tests would be to detect those banks that are likely to jointly default and to evaluate the impact of these joint defaults on the banking system. With this tool, we can also assess how they may lead to cascade failures and losses amplification.

We develop a methodology to detect bank communities that are likely to default jointly. While the main literature stream makes efforts to finding the systemically important financial institutions (SIFI) according to some centrality criterion (Papadimitriou et al. (2013)), here we devise a strategy that relies on the identification of bank communities that potentially can inflict large losses to the financial system. We employ a community detection algorithm to uncover these communities with non-negligible joint default probabilities and show that they have larger systemic risk impacts than those provoked by large banks. This suggests that, although size and interconnectedness matters, it is crucial to evaluate the emergence of these kinds of bank communities, which may also be a trigger for systemic risk. Moreover, the identification of whether or not these bank communities exist stands as an important practical task, as bank communities with high pairwise joints DPs may turn the occurrences of joint bank defaults from rare to probable events.

Our third contribution is the development of a new systemic risk measure for the banking system: the systemic stress amplification. While the expected stress measures the additional stress that results from the combination of default events (1, 2 and 3-bank simultaneous defaults) weighted by their probability, the systemic stress amplification identifies the general condition of the capitalization of the banking system, providing an information of additional stress per unit of initial shock. We note that the measure responded coherently to liquidity issues related to foreign capital inflows and bank reserves regulation, in 2010 and 2012.

The recent financial crises have provided evidence for the following contagion channels: risk concentration channel, in which a significant number of banks is exposed to a common risk factor; balance sheet contagion channel, in which failure on debt repayment causes the write off of these assets by the corresponding creditors; price-mediate contagion, related to asset fire sales, that induces losses due to marking-to-market or to having to sell assets that are being fire-sold; and the occurrence of illiquidity spirals, due to margin calls or short-term liabilities. Though these contagion channels can potentially propagate large losses, we focus on the balance sheet contagion channel in this paper. In addition, our framework only deals with solvency issues. Hence, we do not take into account the liquidity of the institutions. Despite this restriction, this approach has the advantage of measuring the stress levels of the entire financial system or of individual institutions. This positive property gives room to the analysis of sources of stress in the financial system, as well as to the identification of groups formed by SIFIs. We can use the proposed methodology as an auxiliary tool for monitoring the financial system, indicating the overall and local stress levels. Given this information, a central bank would have more data to properly take actions to remedy the observed stress situation.

The paper proceeds as follows. In Section 2, we present a literature review on contagion and systemic risk models, emphasizing the network-based ones. In Section 3, we discuss the main contribution of this paper: the systemic risk framework. In Section 4, we provide meta-information on the supervisory and accounting data employed in this paper. In Section 5, we present the main results and findings. Finally, in Section 6, we draw some conclusions.

2 Literature

Contagion is a key factor for systemic risk. We verify a growing literature on contagion between FIs, addressing its theoretical foundations (Allen and Gale (2001); Rochet and Tirole (1996)), the identification of different contagion mechanisms for several markets (Degryse and Nguyen (2007); Elsinger et al. (2006a); Lehar (2005); Mistrulli (2011)), the proposal of models to analytically gauge systemic risk (Catullo et al. (2015); Elsinger et al. (2006a); Iori et al. (2006); Martínez-Jaramillo et al. (2010); van den End (2009)), the design of empirical tests (Castiglionesi (2007); Hsiao et al. (2011); Pe et al. (2010)) and the investigation of methods for preventing or minimizing contagion (Castiglionesi (2007)).

Allen and Gale (2001) is a seminal paper that models financial contagion as an equilibrium phenomenon. They find that a small liquidity preference shock in one region can spread by contagion throughout the economy. Allen and Gale (2001) argue that the possibility of contagion strongly depends on the completeness of the structure of interregional claims, being more robust when complete.

Following that, Allen and Gale (2004) show that a large number of possibilities exists concerning the relationship between market structure and financial stability. Since these are important arguments and, as there are trade-offs between these aspects of the banking system, prudential regulatory intervention and supervision are needed in their various forms.

Contagion has also been measured more broadly by taking into account different shocks. Elsinger et al. (2006a) simulate the joint impact of interest rate shocks, exchange rate shocks and stock market movements on interbank payment flows of Austrian banks. They distinguish between insolvency due to correlated exposures and due to domino effects. They also show that both correlation and interlinkages are important determinants for assessing systemic financial stability. In particular, the probability of systemic events, such as the joint breakdown of major institutions, is underestimated when we ignore cross-correlations between banks.

A review and criticism of the literature lies in Upper (2007), a survey paper that analyzes computer simulations of contagion due to interbank lending, gathering information on the methodologies they use. He emphasizes that one has to bear in mind the potential bias caused by the very strong assumptions underlying these simulations. Those papers suggest that contagion due to lending in the interbank market is likely to be rare. However, if contagion does take place, the costs to the financial system can be very high, destroying a sizable proportion of the banking system in terms of total assets. He also verifies that the majority of the simulations available in the literature is based on models that only incorporate extremely rudimentary behavior by banks or policymakers. Besides, he stresses the need of taking into account multiple shocks (many simulations usually begin with a shock in a single bank), as in Elsinger et al. (2006a), and highlights the relevance between insolvency and illiquidity of banks, such as in Müller (2006).

Another literature stream that has received increasing attention is of network theory analysis applied to financial markets. In this respect, Inaoka et al. (2004) analyze the network structure of financial transactions, using data of financial transactions through the Bank of Japan payments' system. They find evidence of a certain degree of robustness within the network and suggest an analysis of its dynamics. A similar work is performed in Iori et al. (2008), who analyze the Italian segment of the European overnight money market through methods of statistical mechanics applied to complex networks. Reinforcing the same topic, Soramäki et al. (2007) also investigate the network topology of the interbank payments transferred between commercial banks over the Fedwire Funds Service (USA), finding that the network properties changed considerably in the immediate aftermath of the events of September 11, 2001. Moreover, Embree and Roberts (2009) describe the daily and intraday network structure of payment activities in the Canadian Large Value Transfer System, finding that there are only few stable systemically important participants. In a related approach, Tabak et al. (2013) investigate how bank sizes and market concentration affect performance and risk buildup in networks formed by several Latin American countries.

The influence of the network topology on determining systemic risk levels in financial systems is another line of research that has obtained increased attention by academics and policymakers. In this aspect, Nier et al. (2007) measure systemic risk by varying network formation parameters that define the structure of the financial system, such as level of capitalisation, the degree to which banks are connected, the size of interbank exposures and the degree of concentration of the system, and analyse the influence of these parameters on the likelihood of knock-on defaults. In turn, Battiston et al. (2012a) show that financial networks can be more resilient for intermediate levels of risk diversification rather than for extreme values. Teteryatnikova (2014) highlights the advantages of tiered networks by analyzing risk and potential impact of system-wide defaults in a tiered banking network, in which a small group of head institutions has many credit linkages with other banks, while the majority of banks has only a few links.

Several papers also propose suitable network measurements that serve as indicators to estimate systemic risk. In the financial network literature, Battiston et al. (2012b) proposes the DebtRank measure, which is a feedback centrality measure, to estimate financial stress in the network. In turn, Silva et al. (2015) introduce network measurements that identify remotely vulnerable and contagious FIs, which are useful indicators for bank monitoring and surveillance. Martínez-Jaramillo et al. (2014) provide network measurements related to systemic risk for the Mexican banking system in a systematic and comprehensive manner. Generally, these indicators belong to a class of measures called centrality measures, which is a topic extensively explored in the complex network literature (Silva and Zhao (2016)). Among some classical centrality measures, we highlight Katz and Bonacich indices, PageRank, communicability, betweenness.

3 Methodology

In this section, we present the two main contributions of the paper.

In Section 3.1, we describe our framework to evaluate systemic risk. The model weighs the financial stress that an initial shock scenario causes on the system to the probability of occurrence of that scenario. To estimate the occurrence of these scenarios, we evaluate the DPs of individual, pairs, and groups of three FIs. We do not consider scenarios with higher orders of joint defaults due to data unavailability. To evaluate the expected systemic risk of the financial system, we combine all of these initial shock scenarios and their corresponding stress values. To prevent double counting in this process, we design initial shock events that are disjoint.

In Section 3.2, we provide a novel heuristic for designing feasible and relevant stress-testing scenarios. The procedure identifies bank communities in a graph constructed from pairwise default probabilities. We construct this graph by keeping only those connections that lead to large stress levels in the financial system. By doing this, we retrieve stress-testing scenarios that are relevant from a financial surveillance viewpoint. In addition, by identifying strongly-connected bank communities, we filter out unfeasible stress-testing scenarios.

3.1 Systemic risk

In Section 3.1.1, we briefly review the DebtRank methodology that we use to estimate financial stress in the network given an initial shock. In Section 3.1.2, we detail the framework to compute the expected systemic risk of the financial system. Finally, in Section 3.1.3, we define a novel systemic risk indicator that gauges the expected stress amplification of the financial system.

3.1.1 DebtRank

We employ the DebtRank method to compute the additional stress that occurs in the financial system due to an exogenous initial shock on one or more FIs. Throughout this paper, we use the terms "impact" and "DebtRank" interchangeably.

The computation of the DebtRank measure is inspired by feedback centrality measures. Feedback centrality measures are those in which the centrality of a node depends recursively on the centrality of its neighbors.

The intuition associated with DebtRank is as follows. Suppose we have a network of mutually exposed banks. Each of these banks has assets and liabilities, among which a fraction is related to the counterparties within the network, and a capital buffer. If a bank suffers assets losses greater than its capital buffer, it becomes insolvent and we assume it defaults. In this case, the bank will not be able to honor any of its liabilities. If those losses are lower than its capital buffer, the bank will be in distress and will not pay its creditors a proportional part of its liabilities. This very definition of the DebtRank characterizes it as a stress measure: if a bank suffers a loss of 90% of its capital buffer, it is solvent and does not propagate any losses in a default cascade methodology, such as in that defined in Eisenberg and Noe (2001). In the DebtRank method, however, the bank is almost insolvent and propagates losses by paying only 10% of its liabilities for the network members. We can say that losses are potential in the DebtRank method. Therefore, only a portion or none of them may materialize at all. The creditors of the defaulted bank, in turn, will suffer losses and undergo through the same process. This feedback process continues until the system converges.

The DebtRank method (Battiston et al. (2012b)) models the interbank market as a directed network $\mathscr{G} = \langle \mathscr{V}, \mathscr{E} \rangle$, in which FIs compose the vertex set $\mathscr{V}$ and the exposures between them compose the set of edges $\mathscr{E}$. These links are represented by a weighted adjacency matrix $\mathbf{A}$, where the (i, j)-th entry, $\mathbf{A}_{ij}$, denotes the amounts lent by institution i to institution j. The total liabilities of i are given by $L_i = \sum_{j \in \mathscr{V}} \mathbf{A}_{ji}$ and the relative economic value of an institution i is given by $v_i = L_i / \sum_{i \in \mathscr{V}} L_i$, which is the fraction of i's liabilities with respect to the total liabilities in the interbank market. Our option contrasts with that of Battiston et al. (2012b), who define the relative economic value of an institution as the share of i's assets to the total assets in the network. However, we use the liabilities share because, once a banks default, the losses that other members in the network have correspond to the liabilities of that defaulted bank towards them.

Each institution i has a capital buffer against shocks, E_i, which is its tier 1 capital. If $E_i \leq \gamma$, the firm defaults, where γ is a positive threshold. If vertex j defaults, all of the neighbors i will suffer losses amounting to their exposure towards j, given by $\mathbf{A}_{ij}$. When $\mathbf{A}_{ij} > E_i$, then vertex i defaults. The local impact of j on i is $\mathbf{W}_{ji} = \min\left(1, \mathbf{A}_{ij}/E_i\right)$ so that if i's losses exceed its capital, the local impact is 1.

The presence of cycles in the network inflates the computed impacts by counting the local impact of a node onto another more than once. To avoid the distortion caused by this double-counting, Battiston et al. (2012b) present an algorithm that allows a single impact propagation per each node. We define the state of FI i as being described by the following dynamical variables at time t:

- $h_i(t) \in [0, 1]$, which accounts for the stress level of i. If $h_i(t) = 0$, i is undistressed; when $h_i(t) = 1$, i is on default.

- $s_i(t) \in \{U, D, I\}$, which is a categorical variable that assumes one of the following values: undistressed (U), distressed (D) and inactive (I).

DebtRank evaluates the additional stress caused by some initial shocks using a dynamical system. We model these shocks by adjusting the initial conditions of that system when $t = 1$. The institutions with initial stress level $h_i(1) = 0$ are undistressed, i.e., $s_i(1) = U$. If $h_i(1) > 0$, they are distressed, i.e., $s_i(1) = D$. Those institutions with $h_i(1) = 1$ are initially on default. We update this dynamical system using the following update rules:

$$h_i(t) = \min\left\{1, h_i(t-1) + \sum_{j \in \mathscr{V}} \mathbf{W}_{ji} h_j(t-1)\right\}, \text{ where } j \mid s_j(t-1) = D,$$

$$s_i(t) = \begin{cases} D & \text{if } h_i(t) > 0; s_i(t-1) \neq I \\ I & \text{if } s_i(t-1) = D \\ s_i(t-1) & \text{otherwise} \end{cases} \tag{1}$$

In (1), the dynamical variables $h_i(t), \forall i \in \mathscr{V}$, are updated using available information from the previous step. After this, the variables $s_i(t)$ are also updated. If $s_i(t-1) = D$, $s_i(t) = I$, preventing institution i of propagating impact to its successors more than once.

After a finite number of steps T, the dynamics stops. We compute the resulting DebtRank due to the initial shock scenario $h(1)$ as follows:

$$\begin{aligned} DR &= \sum_{j \in \mathscr{V}} h_j(T)\nu_j - \sum_{j \in \mathscr{V}} h_j(1)\nu_j. \\ &= \sum_{j \in \mathscr{V}} h_j(T)\nu_j - S, \end{aligned} \tag{2}$$

in which S denotes the impact of the initial shock:

$$S = \sum_{j \in \mathscr{V}} h_j(1)\nu_j. \tag{3}$$

Note that DR in (2) effectively evaluates the additional stress in the network that results from the initial shock $h(1)$.

3.1.2 Expected systemic stress

To compute the expected systemic stress, we first need to evaluate DPs of individual, pairwise, and groups of three FIs. We use Merton (1974)'s structural model to evaluate the individual DPs and Segoviano (2006)'s CIMDO methodology to estimate the pairwise DPs.[1] In contrast, we propose a new procedure to evaluate DPs of groups of three FIs.

We assume that these DPs are not affected by the impact propagation process, that is, changes in cross-exposures of banks occurring in a short time span do not affect DPs.[2]

[1] Appendix A details how to evaluate individual DPs from the Merton (1974)'s structural model. In addition, Appendix B shows the underpinnings of the Segoviano (2006)'s CIMDO methodology that we employ to calculate pairwise DPs.

[2] In this paper, we consider that DPs are probabilities that a bank' assets value falls below its distress barrier in a 1-year time horizon, starting from the current value and following a stochastic process. Considering

We avoid double-counting of individual banks' expected impacts by considering a sample space $\mathscr{D}$ composed of tuples that represent all possible combinations of occurrences of idiosyncratic defaults in the banking system. We associate each tuple with a joint default probability measure together with the impact it causes should that probability event occurs as an initial shock scenario. Since we deal with all of the possible combinations of bank default events, the overall probability must sum up to 1. Moreover, these tuples are all pairwise disjoint events. Thus, we can compute the expected systemic stress as the sum of products between the DPs and the impact associated with the default events represented by each of the tuples in $\mathscr{D}$.

Let I be a random discrete-valued variable, which we associate with impacts I_{E_i} that arise from default events of single banks $E_i \in \mathscr{D}$. Initially, consider only two events E_1 and E_2 that we associate with the defaults of banks 1 and 2, respectively. The distribution of I is:

$$I(j) = \begin{cases} I_{E_1}, & \text{if } j \in E_1 \setminus E_2 \text{ tuple.} \\ I_{E_2}, & \text{if } j \in E_2 \setminus E_1 \text{ tuple.} \\ I_{E_1 \cap E_2}, & \text{if } j \in E_1 \cap E_2 \text{ tuple.} \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

Then, the expected impact from these defaults is:

$$\begin{aligned} \mathbb{E}[I] = & \sum_{j \in \mathscr{D}} X(j)p(j) \\ = & I_{E_1} P(E_1 \setminus E_2) + I_{E_2} P(E_2 \setminus E_1) + I_{E_1 \cap E_2} P(E_1 \cap E_2) \\ = & I_{E_1} \left[P(E_1) - P(E_1 \cap E_2)\right] + I_{E_2} \left[P(E_2) - P(E_1 \cap E_2)\right] \\ & + I_{E_1 \cap E_2} P(E_1 \cap E_2) \end{aligned} \tag{5}$$

We note that we cannot sum both regions from events E_1 and E_2 and subtract that from $E_1 \cap E_2$ because each region has its own weight I. Equation (5) means that in the region of the sample space in which only bank 1 defaults, impacts are I_{E_1}. Similarly, in the region in which only bank 2 defaults, impacts are I_{E_2}. Finally, in the region where banks 1 and 2 both fail, impacts are $I_{E_1 \cap E_2}$.

Generalizing for n events, we can calculate the expected impact of the system as follows:

that interbank exposures are a part of the bank's assets, we make a simplifying assumption that interbank exposures changes are included in bank assets changes and, therefore, do not produce additional effects on DPs.

$$\mathbb{E}[I] = \sum_{i=1}^{n} X_i^{(1)} P(E_i) + \sum_{1 \le i < j \le n} X_{i,j}^{(2)} P(E_i \cap E_j) + \\ + \sum_{1 \le i < j < k \le n} X_{i,j,k}^{(3)} P(E_i \cap E_j \cap E_k) + \dots \tag{6}$$

with:

$$\begin{aligned} X_i^{(1)} &= I_{E_i}, \\ X_{i,j}^{(2)} &= I_{E_i \cap E_j} - I_{E_i} - I_{E_j}, \text{ and} \\ X_{i,j,k}^{(3)} &= I_{E_i \cap E_j \cap E_k} - I_{E_i \cap E_j} - I_{E_i \cap E_k} - I_{E_j \cap E_k} + I_{E_i} + I_{E_j} + I_{E_k}. \end{aligned} \tag{7}$$

The time for computing $\mathbb{E}[I]$ grows exponentially as the number of events or banks increases. For two events, for instance, we need to compute impacts I_{E_1}, I_{E_2} and $I_{E_1 \cap E_2}$. In contrast, for n events, we have to evaluate $2^n - 1$ terms, which is computationally infeasible for a real financial system that often has large n. Moreover, due to data unavailability, we cannot calculate the joint probabilities of three or more simultaneous default events. To circumvent this limitation, we approximate the expected impact for n events using only the contributions of single, pairs, and groups of three banks. They correspond to the first three terms in the RHS of (6).

Since we do not have the exact default probability of bank triples, we estimate $P(E_i \cap E_j \cap E_k)$ using proxies. Now, we focus on the employed methodology to obtain these proxies $P(E_i \cap E_j \cap E_k)$, $\forall i, j, k \in \mathscr{V}$, to allow for the computation of the expected systemic stress using up to three intersection terms.

First, we rewrite (6) in a way to represent the union probability of default events in the financial system using the inclusion-exclusion principle:

$$P\left(\bigcup_{E \in \mathscr{D}}\right) = \sum_{i=1}^{n} P(E_i) - \sum_{1 \le i < j \le n} P(E_i \cap E_j) + \\ + \sum_{1 \le i < j < k \le n} P(E_i \cap E_j \cap E_k) - \cdots \tag{8}$$

Our estimations consist in finding the maximum and minimum bounds for $P(E_i \cap E_j \cap E_k)$, given that we know the marginal and pairwise DPs. The upper limit is given by:

$$P_{up}(E_i \cap E_j \cap E_k) = \min(P(E_i \cap E_j), P(E_i \cap E_k), P(E_j \cap E_k), 1 - P_{res}(\mathscr{S}_3)), \tag{9}$$

in which $P_{res}(\mathscr{S}_3)$ is computed for the set $\mathscr{S}_3$ formed by the events E_i, E_j and E_k according to (10), here defined for a generic set of g events $E_1, \cdots, E_g$:

$$P_{res}(\mathscr{S}_g) = \sum_{i=1}^{g} P(E_i) - \sum_{1 \le i < j \le g} P(E_i \cap E_j). \tag{10}$$

The first three terms in the $\min(\cdot)$ operator in (9) come from the fact that a 3-event intersection cannot be greater than the minimum of pairwise intersections of the same events. The last term guarantees that $P(E_i \cup E_j \cup E_k) \le 1$.

We consider that the lower limit is zero:

$$P_{low}(E_i \cap E_j \cap E_k) = 0 \tag{11}$$

We propose that the union probability will stay in the interval given by: $[P_{res}, P_{res} + \sum_{i,j,k} P_{up}(E_i \cap E_j \cap E_k)]$. The reasoning behind this is that each term i of (8) overestimates the probability of the union of the intersection of i events by considering them as disjoint ones. Thus, the term $i+1$ corrects this surplus by taking into account the intersections of $i+1$ events with the opposite sign. For instance, the first term of the equation considers that the union probability is the sum of n single (disjoint) events. If they are not, the second term subtracts the sum of all intersections of two events, considering them as disjoint ones, thus, the third term adds the sum of intersections of three events, and so on.

Writing (8) as the sum of its three first terms and substituting in the third term the upper or the lower bounds of the 3-event intersection probabilities may yield results outside the interval $[0,1]$. That is possible because if we only write the first i terms of (8), we are considering that the terms from $i+1$ onwards are zero. Probabilistically speaking, this hypothesis is the same as considering that high-order intersections are disjoint (hence zero), which is a rather strong assumption.

To correct this caveat using the available information, we estimate a medium term from the upper and lower bounds for each of the DPs of bank triples in the summation in the third term of (8), such that substituting those terms back into equation we achieve a probability inside the unitary interval. Specifically, we estimate the medium term as:

$$\begin{aligned} P_{med}(E_i \cap E_j \cap E_k) = {} & P_{low}(E_i \cap E_j \cap E_k) + \\ & + \alpha(P_{up}(E_i \cap E_j \cap E_k) - P_{low}(E_i \cap E_j \cap E_k)) \end{aligned} \tag{12}$$

in which α is a proportionality factor. Note that we use the same proportionality factor

α when computing the medium terms of all of the 3-event intersection probabilities. We express α as a ratio in which the numerator is the difference of the summation of the lower limits of $P(E_i \cap E_j \cap E_k)$ to the residual formed by the sum of the joint default probabilities of order higher than two in (8) and the denominator is the difference between the summations of upper and lower limits of $P(E_i \cap E_j \cap E_k)$:

$$\alpha = \frac{P(E_1 \cup \ldots \cup E_n) - P_{res}(\mathscr{S}_n) - \sum_{1 \leq i < j < k \leq n} P_{low}(E_i \cap E_j \cap E_k)}{\sum_{1 \leq i < j < k \leq n} \left[P_{up}(E_i \cap E_j \cap E_k) - P_{low}(E_i \cap E_j \cap E_k) \right]}, \tag{13}$$

We also approximate the union probability of all default events by the maximum 3-event union probability, due to data unavailability:

$$P(E_1 \cup \ldots \cup E_n) = \max_{E_i, E_j, E_k \in \mathscr{D}} P(E_i \cup E_j \cup E_k). \tag{14}$$

Substituting (14) and (11) into (13) we get:

$$\alpha = \frac{\max_{E_i, E_j, E_k \in \mathscr{D}} P(E_i \cup E_j \cup E_k) - P_{res}(\mathscr{S}_n)}{\sum_{1 \leq i < j < k \leq n} P_{up}(E_i \cap E_j \cap E_k)}. \tag{15}$$

We conclude the computation of the expected systemic stress using (6), substituting the probabilities in the third summation term by $P_{med}(E_i \cap E_j \cap E_k)$ as computed in (12).

3.1.3 Systemic stress amplification

DebtRank measures the additional stress in the financial system that results from a given initial shock. In this paper, we mold initial shocks as idiosyncratic defaults of individual FIs and pairs and triples of financial institutions. We can measure the stress amplification related to a given initial shock S by computing the ratio of the additional stress to the initial shock. The intuition related to the measure is that, given an initial shock, the additional stress inflicted on the financial system will be greater if, *ceteris paribus*, the capital buffer is lower. Based on this measure of stress amplification, we define the systemic stress amplification (SSA), a systemic risk measure for the banking system, as the ratio of the expected systemic stress to the systemic initial shocks given by:

$$SSA = \frac{\mathbb{E}[I_{E_1 \cup \cdots \cup E_n}]}{\mathbb{E}[S_{E_1 \cup \cdots \cup E_n}]} \tag{16}$$

Above, $\mathbb{E}[I_{E_1 \cup \cdots \cup E_n}]$ is the expected systemic stress computed as proposed in the previous section when we assume that the impact weights in (7) are given by the DebtRank measure *DR*. In contrast, $\mathbb{E}[S_{E_1 \cup \cdots \cup E_n}]$ is the expected systemic idiosyncratic default, computed as the expected systemic stress, but substituting the impact terms in (7) by the corresponding initial shocks as defined in (3).

3.2 Systemic group surveillance

Financial surveillance has become an important topic in the agenda of regulators in view of the recent worldwide crises (Espinosa-Vega and Solé (2011)). It stands as a set of supervisory and regulatory activities that aims at fostering an integrated view of financial sector risks and at improving risk identification and policy analysis. The surveillance of financial institutions permits the effective operation of interbank systems and thus provides tools for assessing their current stability conditions.

We find several works in the literature (Castro and Ferrari (2014); Elsinger et al. (2006b); Zhou (2010)) that focus on the identification and surveillance of systemically important financial institutions (SIFIs). However, little attention has been given in studying the implications of failures of other institutions, not necessarily SIFIs nor large ones, in the financial system stability. Maybe one of the factors that contribute to limiting or focusing surveillance efforts on SIFIs or large institutions comes from the evidence that, in general, defaults of individual institutions that are not SIFIs nor large do not cause significant harm to the financial system. In this work, we show that the existence of bank communities, whose members are strongly interconnected in terms of pairwise joint DPs, may cause significant impact on the financial system when we collectively analyze the default of the community members. We also verify that the great majority of the members in these communities are non-large banks. In this way, this finding suggests that financial stability could be benefited with the broadening of surveillance tools to these bank communities other than to the usual SIFIs and large banks.

3.2.1 Overview

We propose a methodology to design feasible and systemically important stress-testing scenarios. Regulators can use these scenarios in stress-testing schemes that in turn can support the surveillance of the financial system. The idea is to find groups or communities of FIs that can potentially inflict significant amounts of stress in the financial system. The reason we use a collective view of financial institutions instead of a local approach is because the network can amplify initial shocks. Therefore, a joint default of a subgroup of vertices can introduce much more financial stress in the system than if we evaluate that from individual shocks on each of these members. In this way, local approaches would not be able to identify these communities.

We intend to identify groups or communities of FIs that can jointly default and also to compute their corresponding systemic additional stress and risk amplification. The purpose of this methodology is to answer the questions: Are there in the banking system groups of FIs that could jointly default? If so, which are these groups? How significant is the impact that those default events would produce? To answer these questions, we first identify communities of FIs that could most probably default together. Then, we compute the additional stress, using the DebtRank method, and the systemic risk amplification caused by the joint default of members of these communities.

We start by forming an undirected weighted network $\mathscr{G}^{(DP)} = \langle \mathscr{V}^{(DP)}, \mathscr{E}^{(DP)} \rangle$, where $\mathscr{V}$ and $\mathscr{E}$ are the sets of vertices and edges, respectively. The vertex set $\mathscr{V}^{(DP)}$ is identical to that of the exposures network, in which we represent its elements by financial institutions. Initially, we create an edge between $i \in \mathscr{V}^{(DP)}$ and $j \in \mathscr{V}^{(DP)}$ with weight according to $P(E_i \cap E_j)$, that is, the pairwise joint DP of i and j. As the DP of joint default increases, the larger will be the edge weight linking both entities. Though pairwise default probabilities have often small values, they are still positive, meaning that we will have a near-complete network structure. We describe the resulting network topology using the weighted adjacency matrix of $\mathscr{G}^{(DP)}$, which we denote here as $\mathbf{A}^{(DP)}$.

Since edges carry information about how likely pairs of institutions can jointly default, the intuition of our methodology for group surveillance is to find and monitor those bank communities whose members are highly interconnected and, at the same time, have few links with the remainder of the network. Since members of the same community are expected to have several links with large weights connecting each other, the default of the community as a whole is a feasible event to occur. To identify these communities, we employ a community detection procedure that works in a networked environment.

Community detection is an unsupervised learning procedure that has received increasing attention by the community due to its applicability in various branches of science (Silva and Zhao (2012, 2013)). Community detection algorithms rely on certain assumptions over the network structure to perform well in the identification of communities. One of them states that the existence of structural and well-defined communities is only possible when graphs are sparse (Fortunato (2010)). Sparseness arises when the number of edges is of the order of the number of vertices. If the network is dense, the distribution of edges among the vertices is too homogeneous for communities to make sense. Thus the network structure is unlikely to convey relevant information to identify the community structures. Recalling that our network structure is highly dense, we need to employ a sparsification procedure, such as to transform the resulting network in a sparse graph, while maintaining the most important information about the pairwise default probabilities between institutions. In this sparsification process, we use information on the expected systemic impact of each of the links in the networks. In order to find systemically relevant stress-testing scenarios, we only maintain those links whose expected systemic impact is large.

We opt to use an agglomerative community detection method because it gives us a sense of how strong pairwise connections between members of the same community are, in such a way that we can rank those connections that are at the core of the community and those that are terminal or peripheral to the community. Idiosyncratic shocks on institutions that have core connections are more likely to cascade to other community members, because those institutions are more central in the community.

In the remainder of this section, we first show how we conduct the sparsification procedure such as to guarantee the sparseness of the network and therefore the existence of well-defined communities. Following that, we detail the community detection procedure that we use to identify communities in the graph of pairwise default probabilities.

3.2.2 Network sparsification process

The network that we construct using pairwise joint DPs is likely to be very dense, with the majority of the edge weights corresponding to negligible numbers. Here, we discuss how we apply the proposed sparsification process such as to obtain a sparse network, while retaining the most important network edges from a viewpoint of systemic group surveillance.

One natural approach would be to discard all of those pairwise joint DPs that are smaller than a small threshold. One drawback in using this approach is the potentiality of discarding small joint DPs that ultimately lead to large potential losses in the financial system. In order to circumvent that, we opt to discard edges according to their expected potential loss (**EPL**). The **EPL** of institutions i and j is

$$\mathbf{EPL}_{ij} = \left(\mathbb{E}[I_{E_i \cup E_j}] + \mathbb{E}[S_{E_i \cup E_j}]\right) TA, \tag{17}$$

in which $\mathbb{E}[I_{E_i \cup E_j}]$ and $\mathbb{E}[S_{E_i \cup E_j}]$ are the expected systemic additional stress and idiosyncratic default, respectively, as defined in the previous section. We multiply the total stress by the total interbank assets $TA = \sum_{ij} \mathbf{A}_{ij}$ so as to convert the stress measures into the monetary domain. Thus, $\mathbf{EPL}_{ij}$ gives us a sense of the expected potential loss that would occur in the financial system if institutions i and j jointly default.

From a systemic viewpoint, it is inviable to monitor each feasible link in the network. A good strategy is to only keep track of those links that lead to the largest expected potential losses. By applying a sparsifying process on the network, we are effectively removing those links that have negligible expected potential losses. Thus we filter links of the original weighted adjacency matrix $\mathbf{A}^{(DP)}$ as follows:

$$\mathbf{A}'^{(DP)}_{ij} = \begin{cases} \mathbf{A}^{(DP)}_{ij}, & \text{if } \mathbf{EPL}_{ij} \geq \sigma \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

in which $\sigma \in [0, \infty)$ is user-supplied threshold that delineates the minimum expected potential loss of links that we will consider in the filtered weighted adjacency matrix $\mathbf{A}'^{(DP)}$. Note that we can adjust the network sparseness by varying σ. As σ increases, the network gets sparser. Due to the sparsification procedure, we expect that the resulting graph will only have links that have representative potential losses.

Finally, we compare the approach of this work with several others in the literature that attempt to estimate topological properties of the network using Minimal Spanning Trees (MST) often from the cross-correlation matrix of a portfolio of financial assets (Bonanno et al. (2003)) or credit default swaps (Puliga et al. (2014)), among others. We argue here that the resulting network structure of an MST may not represent in a truthfully manner the communities in network of joint DPs. For instance, say we have a large community with large edge weights, meaning that any pair of banks has high joint DP. If we construct an MST from this graph, we first would lose this dense structure of links, possible creating several different communities, when in fact all of the banks are heavily connected to each other. Second we would end up with a graph with the largest joint DPs that necessarily do not have significant expected systemic stress should the pairs of FIs default. We can say that the MST approach is a very aggressive sparsification process. In fact, it theoretically results in the sparsest connected network structure. In this work, we apply a milder sparsification procedure that attempts to retain only those representative links that not only have high joint DPs but also significant additional stress.

We also point several drawbacks in using cross-correlation information to construct a network. First, the fact two banks have higher than average correlation in normal times does not guarantee to be a good proxy of the correlation of their defaults. If we construct a network from this, community detection algorithms would probably declare banks with highly correlated default probabilities as members of the same community. Second, the cross-correlation is very sensitive to the period range used to compute the DPs. Third, the network structure is highly dependent on the employed correlation index: Pearson, Spearman, etc. We get results that are more robust by using joint DPs in our analysis, as we get rid of the difficult problem of tuning the network using similarity metrics that are based on cross-correlation indices.

3.2.3 Community detection algorithm

In this section, we describe the community detection algorithm that we apply to find bank communities. As input of the algorithm, we use the sparsified weighted adjacency matrix $\mathbf{A}'^{(DP)}$. We use the community detection algorithm proposed by Clauset et al. (2004). This algorithm is a hierarchical community detection method that uses a bottom-up or agglomerative approach. As such, at the initial step, it treats each vertex as a single community and then successively merge pairs of communities until all clusters are effectively merged into a single giant community that contains the graph itself.

The main idea of the algorithm is to optimize a network measurement called modularity. In general terms, the modularity quantifies how good a particular division of a network in communities is. Appendix C provides a formal definition of the modularity. In the community detection algorithm, we monitor how the network modularity evolves as a consequence of community merges. Basically, we declare the best network split as that community configuration in which the modularity is maximal. We are interested in performing systemic group surveillance on that network with optimal modularity.

In the original version, Clauset et al. (2004) apply the method for non-weighted networks. Here, we modify its underlying learning process so as to comport the community detection for weighted networks. For that, we employ the weighted version of the network modularity instead of its binary version.

The motivations for choosing the algorithm of Clauset et al. (2004) are as follows. First, it has become a community detection benchmark in the complex network literature, obtaining good community detection rates both for artificial and real-world networks (Fortunato (2010); Silva and Zhao (2012, 2013)). Second, it does not require parameter adjustments; hence, we can skip model selection. Third, it can find an arbitrary number of communities in the network. These features contrast with several techniques in the complex network literature, some of which designed to only find two communities, others with several parameters to adjust *a priori*, such as the number of communities, their sizes, etc. These kinds of information are usually unknown beforehand. Thus, by using an algorithm that guides its decision based on the data distributions without external information in the form of parameters, we effectively prevent the introduction of biases in the learning process.

Due to its agglomerative nature, Clauset et al. (2004)'s algorithm starts by declaring each vertex as members of a community itself. At each step, the algorithm merges two communities that lead to the largest increase in the modularity Q, i.e., it finds the largest modularity increment ΔQ. In the initial step, the increment in the network modularity if communities i and j are joined is:

$$\Delta Q_{ij} = \begin{cases} \frac{1}{2E} - \frac{s_i s_j}{(2E)^2}, & \text{if } i \text{ and } j \text{ are connected.} \\ 0, & \text{otherwise.} \end{cases} \tag{19}$$

We can stop the merges when the network configuration reaches its maximum modularity using the following heuristics: once we find a negative increment $\Delta Q_{ij} < 0$ during the modularity optimization process, the maximum global value associated to the network modularity has been reached and subsequent merges cannot increase the modularity anymore. This nice property enables us to stop merging communities once we find a negative ΔQ_{ij}. The intuition of choosing the configuration in communities in which the modularity is maximal is because the modularity measure is a proxy of how good the network division is in terms of the existence of communities. Therefore, the network with maximal attainable modularity will have the most well-defined communities in relation to other candidate configurations.

4 Data

Along this paper, we use a unique database with banking supervisory data used by the Central Bank of Brazil.[3] In our analyses, we consider that the banking system is formed by banking conglomerates and individual banking institutions that do not belong to conglomerates, to which we will refer indistinctly as banks[4]. Data from this database comprise banks' individual characteristics, for instance, their capitalization and total assets, and their operations, e.g, the domestic interbank ones.

Our sample has quarterly observations, with end-of-quarter data along the period from March 2010 to December 2014. For each of these dates, we analyze the network of interbank exposures that consists of a directed network in which the vertices are banks[5] and the links between a pair of banks (i, j) are totals of bank i's investments (or exposures) in bank j's unsecured instruments[6]. We aggregate these exposures regardless of the instrument type or time to maturity, as the methodology that we employ only uses information on total exposures. In a directed network, we can have a pair of banks mutually exposed, however, we do not net out these exposures, since compensation is not legally enforceable in the Brazilian jurisdiction. Additionally, for some of the analyses, we categorize banks

[3]The collection and manipulation of the data were conducted exclusively by the staff of the Central Bank of Brazil.

[4]These banks may be universal, commercial, investment or savings ones.

[5]We do not include intra-conglomerate exposures into the analysis.

[6]The most important of these instruments, in volume, are interfinancial deposits, bank deposit certificates, interbank onlending, credit and credit assignment operations, instruments eligible as capital, real state credit bills, financial letters and swap operations, which represent about 95% of the total.

with respect to their sizes (large, medium, small and micro) following the methodology presented in the second Financial Stability Report of 2012 by the Central Bank of Brazil (BCB (2012)).

Table 1 presents average network topological features along the period of analysis. The banking system has, on average, 119 banks. The network is sparse, with a density around 0.057, and presents a structure with money centers. The average network degree, 6.69, is unevenly distributed along banks with different sizes, as can be seen in Fig. 1b, which shows that large banks have a higher average in-degree (number of creditors). The moderately negative assortativity provides evidence that small banks connect preferentially with large ones. This evidence is supported by a small network diameter. A network diameter of 4 means that the shortest path from any bank to any other one has at most 4 steps along the interbank network.

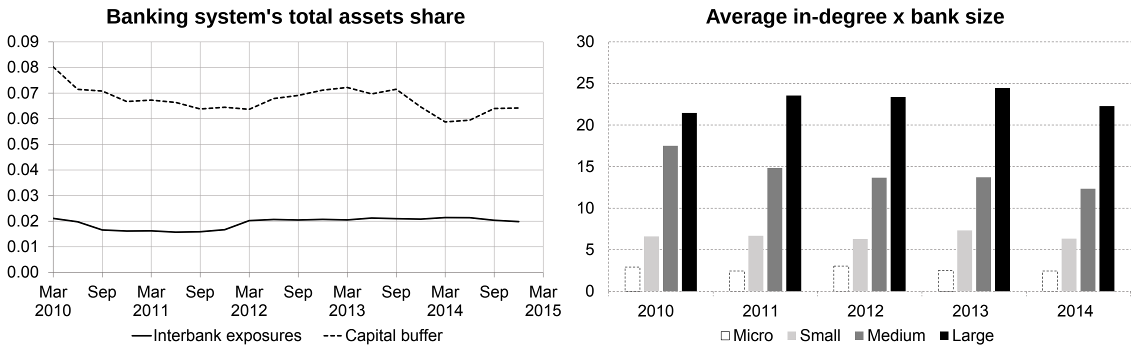
Table 1: *Average network measures from Mar 2010 to Dec 2014.*

Variable	Mean	Std Dev
Number of banks	119	2
Average degree	6.69	0.55
Density	0.057	0.006
Assortativity	-0.37	0.03
Diameter	4	0

Figure 1 also provides information on variables that are relevant to the stress transmission mechanism under study. From a bank-level point-of-view, we compute the stress induced by a loss suffered by a given bank as the ratio of that loss to the bank's capital buffer. Figure 1a shows that, on average, banks are well-capitalized to resist to such losses even if all of their exposures are directed to a single borrower. In this model, the stress suffered by a bank is transmitted to its creditors, that is, a loss suffered by a bank affects its potential ability to repay its debts. Figure 1b shows that large banks propagate stress to a greater number of creditors, which will receive a larger impact if they are less capitalized. However, shocks produced or propagated by large banks can result in comparatively low levels of stress if their creditors have enough capital buffer to absorb them.

5 Results and discussions

In this section, we compute the expected systemic stress of the banking system and its systemic stress amplification from March 2010 to December 2014. To do so, we compute, for each quarter along the sample, the impacts and DPs of individual banks, groups of two and of three of them, and we use (6) and (16) to compute the proposed systemic measures. We also present heuristics to identify communities of banks that are



(a) ***(b)***

Figure 1: *Aggregate interbank exposures (unsecured assets) and capital buffer shares in the banking system's total assets (Figure (a)) and average number of creditors by bank size at the end of each year (Figure (b).)*

most likely to jointly default and compute the expected impact for each of these identified communities. We also relate how these bank communities can support in systemic bank surveillance.

5.1 Evaluation of the financial system stress measures

To compute the proposed systemic risk measures, we first evaluate the DPs in a 1-year horizon of single banks that participate in the interbank market using the Merton structural model as discussed in Appendix A. We use the following parameters:

- *Adjusted total assets* (A).[7]
- *Distress barrier* (DB). Due to the unavailability of time to maturity data of total liabilities (TL), we assume that they are predominantly short-term debts ($STD = 0.7$) with a significant long-term debt ($LTD = 0.3$) share. Assuming[8] $\alpha = 0.5$, which models the proportion of early-redeemable long-term debts, we find that the distress barrier is given by $DB = STD + \alpha LTD = 0.7TL + 0.5 \times 0.3TL = 0.85TL$. For robustness check, we also compare the sum of all single banks expected impacts for $DB \in \{0.8TL, 0.9TL, 1.0TL\}$.
- *Interest rate* (r). We use the interbank interest rate CDI.
- *Assets volatility* (σ_A). We use the annualized standard deviation of the log returns of the adjusted total assets, i.e., $\log\left(\frac{A_t}{A_{t-1}}\right)$.

[7]The adjusted total assets comprises total assets after netting and reclassification of the balance sheet items. Netting is performed on the following balance sheet items: repurchase agreements, interbank relations and relations within branches, foreign exchange portfolio and debtors due to litigation. Reclassifications are made within foreign exchange and leasing portfolios.

[8]The selected value is consistent with the Moody's - KMV *CreditEdge* approach.

- *Horizon* (T). We compute the default probabilities in a 1-year horizon using monthly aggregated balance sheet data. We choose a 1-year time horizon for the reasons pointed in Appendix A.

***Table 2:** Overall expect impact of all single banks. We report the results as a fraction of the total assets of the interbank market. For robustness, we use different values for the distress barrier. Results are reported on a yearly basis using December as the reference month.*

YEAR	DB/TL				
	0.80	0.85	0.90	0.95	1.00
2010	0.0471	0.0554	0.0666	0.0840	0.1138
2011	0.0254	0.0319	0.0403	0.0519	0.0695
2012	0.0189	0.0273	0.0398	0.0585	0.0887
2013	0.0160	0.0232	0.0345	0.0525	0.0831
2014	0.0065	0.0092	0.0134	0.0201	0.0310

Initially, we compare the overall expected impact of all single banks using different ratios of the default barrier to total liabilities. In this way, we can better assess the influence that a particular choice of the DB/TL ratio would have on results. Table 2 reports the results using $DB = 0.80\ TL$ to $1.00\ TL$. For $DB = 0.85\ TL$, the average sum of expected impacts reported in the table is 2.94% of the interbank market total assets. From Table 2, we see that the assumption of higher DB/TL ratios results in much higher expected impacts, with a maximum of 11.38% when $DB = 1.00\ TL$.[9] The larger the short-term or early-redeemable debts are, the larger will be the amount of assets to be fire-sold in case of distress, which, for illiquidity reasons, accelerates increases in losses, and therefore, in DPs. From a numerical point-of-view, the distance to default decreases. As this distance decreases, it becomes easier that a fluctuation on the total assets amount reaches the distress barrier, which can be translated into a higher DP (see (25)).

Next, we compute the expected systemic stress for the banking system in the period from March 2010 to December 2012. Figure 2 presents the upper, medium-term and lower estimates for the expected systemic stress computed for the period using the procedures described in Section 3.1.2. These procedures use DPs computed for single banks and for pairs of them, and estimates for DPs of intersections of default events of three banks, assuming that DPs of 4-bank intersections are zero.

The medium-term estimate is related to a plausible union probability and is closer to the lower estimate. The period of the sample corresponds to the post 2007 global financial crisis. In Brazil, this period began with large foreign capital inflows that resulted from

[9]When $DB = 1.00\ TL$, the FIs' total liabilities are entirely composed of short-term or early-redeemable debts.

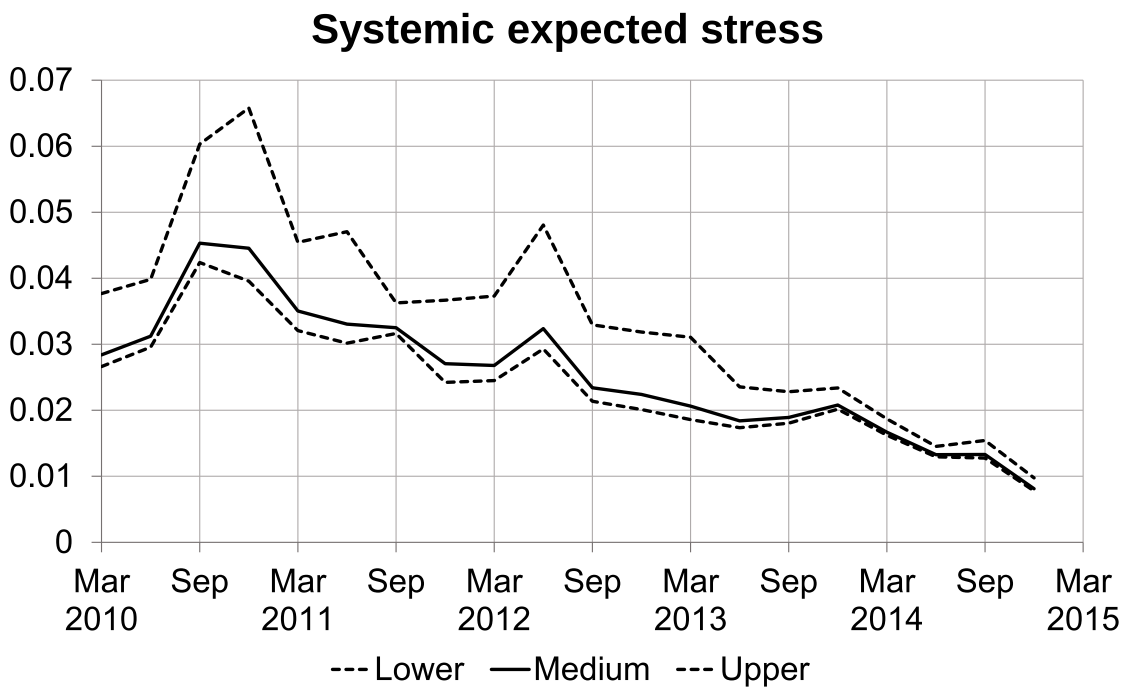


Figure 2: *Expected systemic stress in a 1-year horizon, computed using 3-bank joint-DPs, upper, medium-term and lower estimates.*

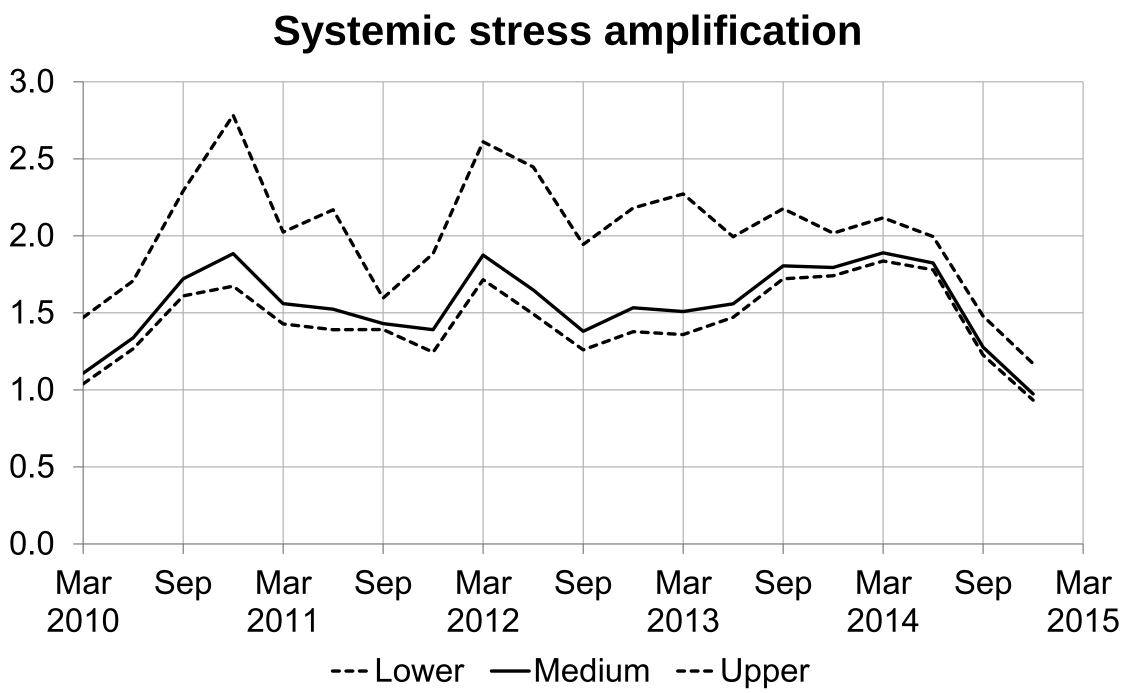


Figure 3: *Expected systemic stress amplification, computed using 3-bank joint-DPs, upper, medium-term and lower estimates.*

the unconventional monetary policies adopted by the US, by one side, and by liquidity issues faced by some of the medium- and small-sized banks, by the other. That increase in the capital inflow raised the liquidity available to banks, that invested in the interbank market, increasing their exposures. However, to counter the liquidity excess, the central bank raised the reserves requirements, which reduced the systemic stress until the end of 2011, when the reserves requirements were lowered to create more favorable conditions to banks that were suffering from lack of liquidity. From September 2012 to the end of the sample, expected systemic stress decreased, the same happening to the boundaries of the interval around the measure. We compare expected systemic stress with systemic stress amplification in Figure 3. Both have peaks in the end of 2010 and in the middle of 2012. However, the amplification grows in the second half of the sample while the expected stress does not.

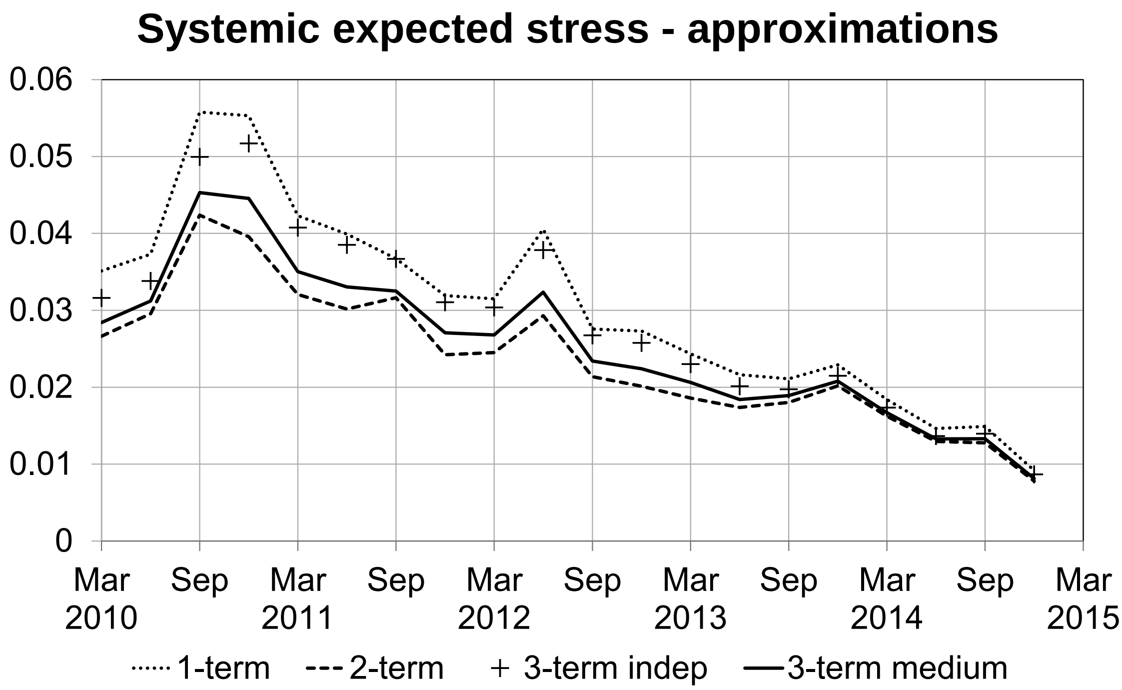
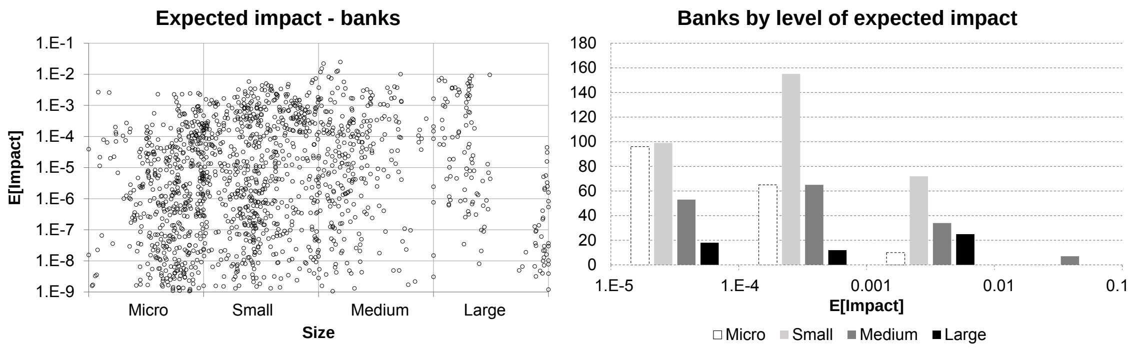


Figure 4: *Approximations for the expected systemic stress in a 1-year horizon, using only the DPs of individual banks (1-term), using also 2-bank joint DPs (2-term), and using additionally 3-bank joint DPs independent-event estimates and medium-term estimates.*

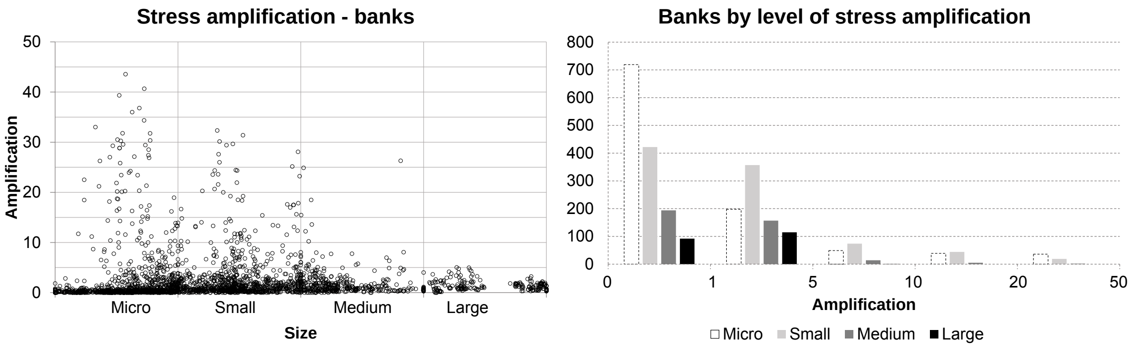
In Figure 4 we compare successive approximations of the expected systemic stress. We present approximations using the first term, the first two terms and the first three terms using the medium-term estimate of the probability of the intersections of three events in the third term of (6). We also compute an approximation with three terms of (6) considering that the banks default events are independent, that is, an approximation in which the probabilities of the intersection of two and three events are computed as the product of the probabilities of the corresponding single events. The chart shows that the 1-term estimate yields the highest results, and that the 2-term one yields the lowest, as expected. The 3-term estimates yield intermediary results. We note that the 3-term independent estimate is overestimated, as the union probability computed from these three terms is way above 1 along all the period. This is a reason for this estimate being superior to the medium-term estimate, which corresponds to a union probability in the interval $[0,1]$. Therefore, given this caveat, it would be necessary to employ the same approach adopted to compute the medium-term estimate of the expected systemic stress, using (12).

Stress amplification, both for single banks and the banking system, measures the additional stress in the banking system that results from a unit of initial shock applied. To see what this measure conveys for different levels of initial shock applied to a single bank or to all the banks in the system, lets suppose initially that we change the sum of the liabilities of a given bank to its counterparties keeping the proportionality between them, and that the liabilities and other characteristics of the other banks remain fixed. Then, stress amplification remains constant, regardless of the shock level, unless there is one or more banks that default. In that case, stress amplification decreases. This is so because stress propagates from a bank i to the bank j exposed to i according to the proportionality factor $\mathbf{W}_{ij} = \min(1, \mathbf{A}_{ji}/E_j)$ (see (1)), thus if bank j defaults due to bank i, $\mathbf{W}_{ij} = 1$ regardless of $\mathbf{A}_{ji}$ being larger or not. Therefore, for a single bank, stress amplification remains constant or decreases as the initial shock increases. As for the systemic stress amplification, as the initial shock applied to each bank of the banking system increases, amplification also increases. This is so because to increase proportionally the initial shock applied to all banks, we must increase all $\mathbf{A}_{ij}$ in the exposures matrix. Figure 3 conveys that, in general, capitalization levels have been kept proportional to bank exposures, at least for the counterparties of banks with largest expected stress. The coincidence of peaks in 2010 and 2012, for figures 2 and 3 convey that they were related to increases in bank exposures. The decrease in Figure 2 after June 2012 is due to decreases in the DPs of the relevant banks, that is, those with highest expected stress.



(a) ***(b)***

Figure 5: *Expected impact for individual banks by bank size (Figure (a)) and bank size distribution by level of expected impact (Figure (b).)*



(a) *(b)*

Figure 6: *Stress amplification for individual banks by bank size (Figure (a)) and bank size distribution by level of stress amplification (Figure (b).)*

We also identify the most relevant banks both for the computation of the expected systemic stress and for the computation of stress amplification. To do that, we gather data on the additional stress related to the default of single banks and on the corresponding stress amplification for all the periods of the sample, showing results in Figures 5a and 6a. Regarding expected impact, we divide the sample into levels according to this variable and find that only medium-sized banks fall into the largest level, whereas in the next lower level, we find mostly small-sized banks. There is a significant share of large-sized banks with expected impacts below $10^{-5}TA$. We explain that by the diversification of investments along different counterparties. Results in Figures 5 and 6 also reflect the distribution of banks from the sample along size categories: large banks are 8.2%, medium-sized, 14.7%, small-sized, 36.1%, and micro, 41.0% of the total number of banks, on average. In Figures 6a and 6b, we see that banks that present the higher stress amplification are small and micro-sized. Although these banks produce a high stress amplification, this is far less relevant than the stress produced by large banks, which have less amplification, but much higher initial shock. A reason for a higher stress amplification could be a mix of low diversification and more leveraged counterparties.

5.2 Systemic group surveillance

In this section, we apply our strategy to identify bank communities in the Brazilian interbank network. These identified communities can potentially serve as stress-testing scenarios and hence can support regulators in the task of financial surveillance.

Since we form the network in which links denote pairwise joint DPs of banks, we first remove those links that do not have significant expected potential losses. Figure 7 illustrates how the network density behaves as we vary the sparsification parameter σ. Note that if we do not apply any filtering procedures, the network density is 86%, density in which the existence of well-behaved communities is not possible. For small values of σ, we already see large declines on the network density, suggesting that the great majority of the links does not convey significant expected potential losses. In the other extreme, we see that 0.4% of the links generate expected potential losses amounting to more than 100 million BRL. Recall that a large or a small expected potential loss between i and j, $\mathbf{EPL}_{ij}$, may be due to suitable combinations of the pairwise joint DP of i and j, $P(E_i \cap E_j)$, and the expected systemic stress of the financial system should i and j default as an initial shock scenario.

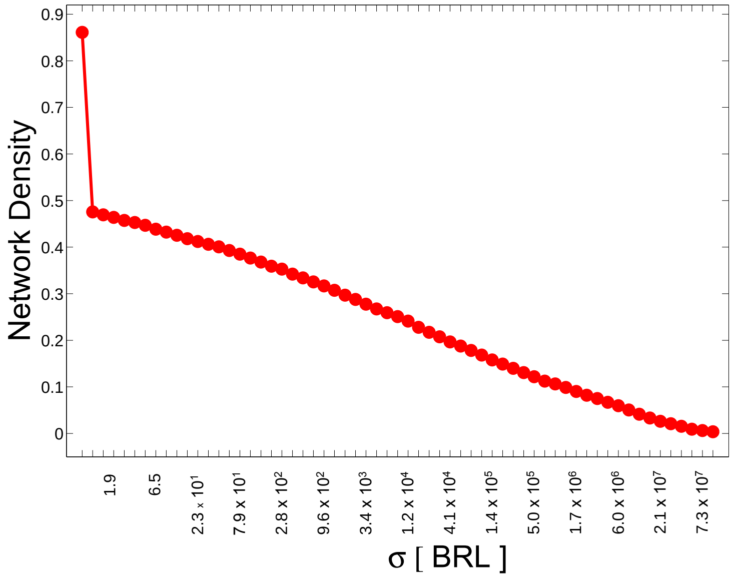
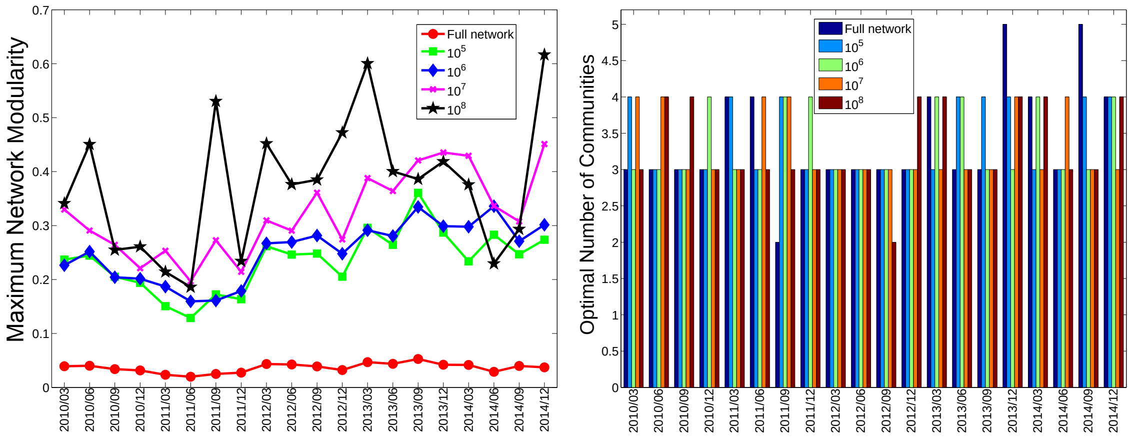


Figure 7: *Behavior of the network density as σ, the sparsification parameter, varies. In the x-axis, we use* 60 *log-spaced values of σ in the interval* $[0, 10^8]$.

Inspecting Fig. 7, we choose the sparsification values $\sigma \in \{10^5, 10^6, 10^7, 10^8\}$, thus assuring sparseness of the network.[10] Figures 8a and 8b portray the maximum modularity of the network and the associated optimal number of communities for the entire studied period when we apply the agglomerate community detection algorithm. Besides the discussed values of σ, we employ the full network without sparsification for robustness. First, it is clear that the full network does not have evident community structures, as the modularity is near zero. Therefore, the community structure in the full network is statistically insignificant and thus resembles a random network. As we increase σ, we start removing the pairwise connections that have small expected systemic impacts. For values of $\sigma \in \{10^5, 10^6, 10^7, 10^8\}$, we can see that the network modularity significantly increases, showing that the community structure becomes apparent and thus the network departs from a random model. Recalling that a modularity greater than 0.3 already gives strong evidence of well-defined communities, we conclude that the Brazilian network has stable community structures in which the members have significant expected systemic stress after 2013. These bank communities are good candidates for a systemic group

[10]As a rule of thumb, we can roughly consider networks as sparse when their densities are below 20%.

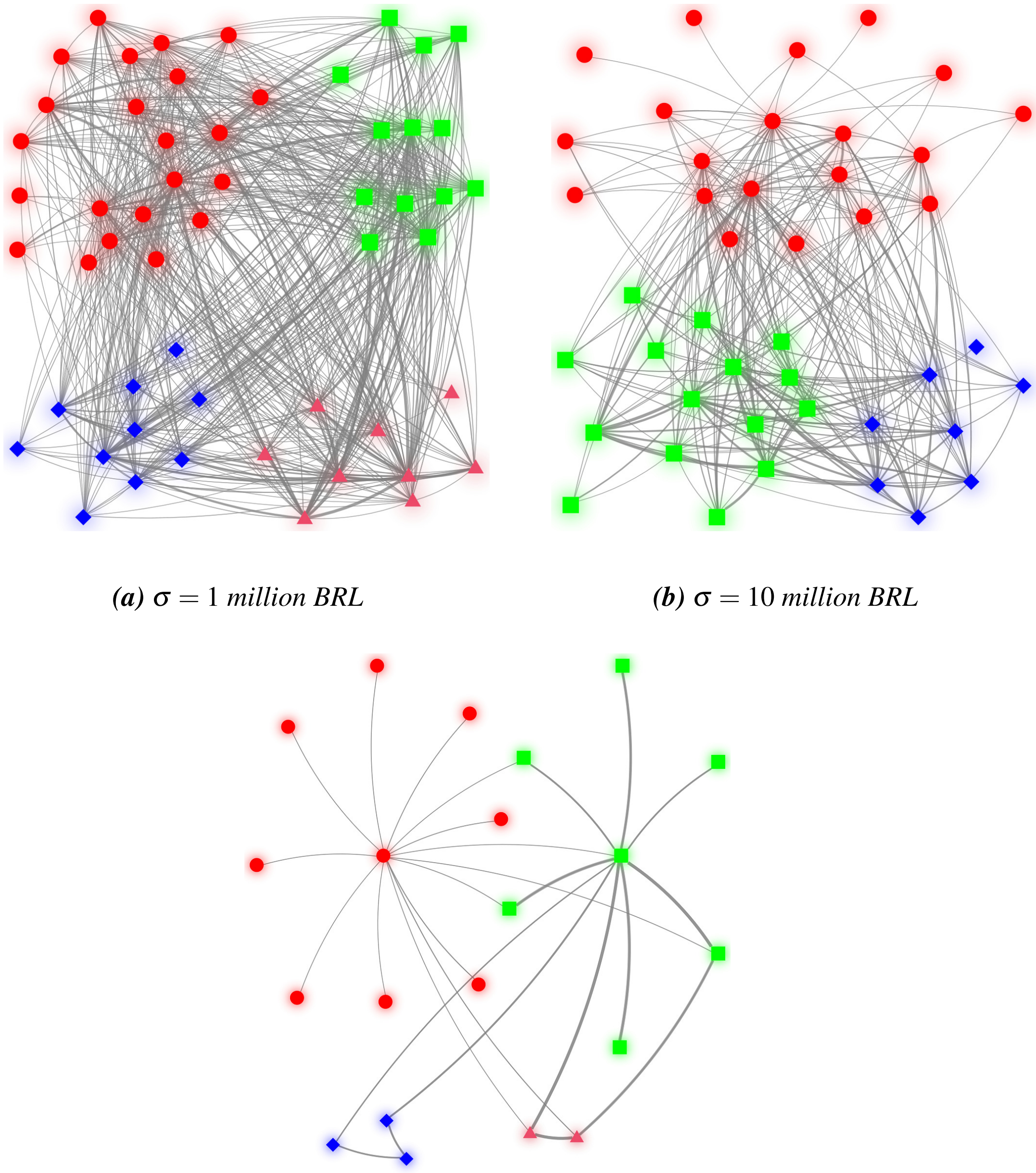
surveillance, because community members are strongly interrelated by significant pairwise joint DPs and also have non-negligible expected potential losses. We also observe that the optimal number of communities in the community detection process is somewhat stable, oscillating roughly between 3 and 4 communities, suggesting a somewhat persistence over time. Considering that we are only maintaining those connections with significant expected systemic losses in the network and that banks engage with few other counterparties to establish their main financial operations possibly due to monitoring costs and relationship lending, we expect that the most important connections in the network will persist over time. Thus, we also expect that the identified communities will be stable.



(a) Maximum modularity *(b) Optimal number of communities*

Figure 8: *Maximum modularity and the associated optimal number of communities in the community detection applied on the sparsified network of pairwise joint DPs. For robustness, we employ several thresholds σ in the network sparsification process.*

In order to get a gist as to how the communities are disposed in the sparsified network of pairwise joint DPs, Figure 9 portrays the network in December 2014 using three different sparsification parameters: $\sigma \in \{10^6, 10^7, 10^8\}$. For clarity, we do not exhibit singleton vertices. Note that the number of vertices with connections effectively reduces as we increase σ. In special, there are 54, 43, and 21 banks when σ assumes the values of 1, 10, and 100 million BRL, respectively. We note that members of the same community share more link weights with each other than with other communities. An interesting feature is of the resulting network topology in Fig. 9c. Besides being clustered in communities as we can see from its significant modularity of 0.62, the network that we sparsify with $\sigma = 10^8$ resembles a scale-free network, with the existence of few hubs and several periphery or terminal vertices. Recalling that links in the network denote probability of joint defaults, this configuration turns out to be potentially harmful as it is one of the networked structures that can quickly spread information to the remainder of the network, as hubs effectively shorten network paths. In contrast, clustered networks that are not scale-free, such as those in Figs. 9a and 9b, information can only efficiently spread among members of the same community, not among those of different communities.



(a) $\sigma = 1$ *million BRL*

(b) $\sigma = 10$ *million BRL*

(c) $\sigma = 100$ *million BRL*

Figure 9: *Visual representation of the community detection results applied on the filtered network of pairwise joint DPs in December 2014. The vertex color or shape denotes its community. We use different thresholds* σ *in the sparsification process and do not exhibit singleton vertices.*

One of the advantages of using an agglomerate community detection algorithm is that we can evaluate how communities are formed. Since in the method of Clauset et al. (2004) we join the pair of communities that gives the maximum modularity increment, those communities that are first joined are central to the resulting community than those ones that are incorporated to the same community later. We can check how these joins take place by inspecting a dendrogram. A dendrogram is a tree diagram employed to illustrate the orderly arrangement of the communities produced by the hierarchical community detection procedure. The vertical axis symbolizes the iterations of the algorithm. In contrast, the horizontal axis shows the communities joined so far. Figure 10 shows the dendrogram of the community detection process when we consider the network in December 2014 with a sparsification parameter $\sigma = 1$ million BRL. At the initial iteration, note that each vertex or bank is a community itself. In the first iterations, the method merges the communities that are formed of banks 22 and 47, respectively. Followed by that, the algorithm again merges the resulting community made up of those two vertices with the community of vertex 23. This process goes on until we are left with a single giant community corresponding to the network itself. From a systemic point of view, it is important to monitor core connections inside the community, because:

- they are the most prone of happening, because they normally encode large joint probabilities between two banks; and
- they have representative expected potential losses.

Returning to our example, in the community formed in the first two steps of the algorithm with vertices 22, 23, and 47, the link between 22 and 47 is more central than that between 23 and that same community. In this way, the dendrogram can effectively rank those connections that should receive more attention in each community. In addition, dendrograms are useful because we can readily partition the network into communities by a horizontal cut at any point in the vertical axis. By drawing a horizontal line at iteration 60, for instance, we get four communities, which is the number of communities for which the modularity is maximal. Figure 8b represents the number of communities that corresponds to the maximum modularity values computed for each date. Therefore, we can easily adjust our systemic view of the network by simply selecting the number of desired communities and by looking at the resulting communities in the dendrogram. For systemic group surveillance, looking at those communities with maximum network modularity is a good strategy, as the corresponding network partition presents more well-defined communities than the other partitions.

We now study the community composition and the additional stress that each one inflicts when the entire community defaults. The task of identifying these communities is already an important feature to regulators narrow their surveillance efforts in the financial system. Here, our focus is on showing the importance that non-large entities play in propagating stress when they are analyzed in a collective manner. We use as baseline the network of joint DPs in December 2014 with $\sigma = 1$ million BRL (see Fig. 9a). Table 3 reports the additional stress caused by each uncovered community by the community detection algorithm. In these filtered networks, most large banks do not appear mainly

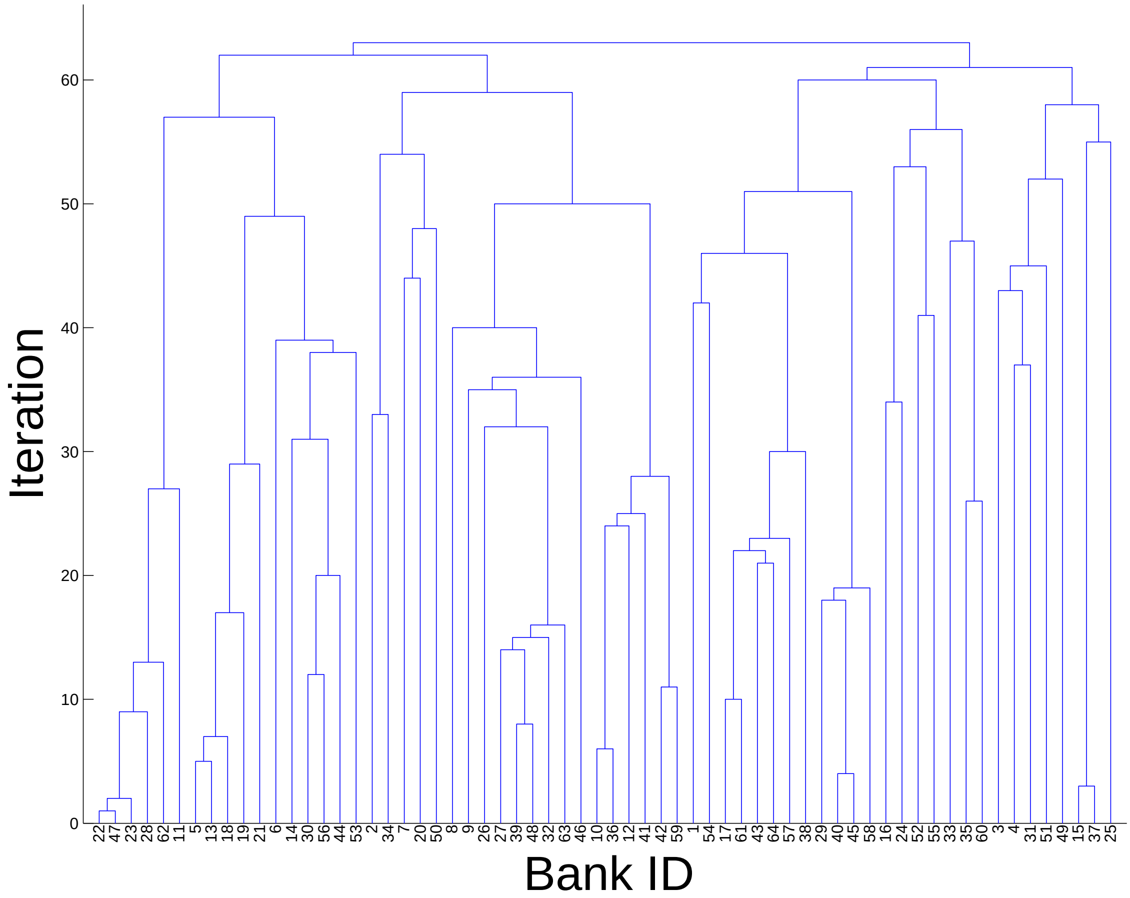


Figure 10: *Dendrogram showing the results of the agglomerative community detection process applied on the interbank network in December 2014 and $\sigma = 1$ million BRL. At the start, every vertex is a community itself. As the iteration progresses, we join those communities in which the increment in the modularity is maximum.*

because their DPs are very small. Thus, during the sparsification process, their financial operations with other counterparties are filtered out as they do not have large expected systemic losses. In special, we only note the existence of three large banks in the community depicted with red circle-shaped vertices. If we simulate a default on these three large banks, we get an additional stress of 0.039. However, if we default all of the members of that same community, including the three large banks, we obtain an additional stress of 0.106. Thus, though the other banks in the community are non-large, they significantly contribute to increasing the additional stress that the financial system can potentially suffer.[11] This point highlights the importance of non-large entities when analyzed in a collective manner. We also note that the community that represents the magenta triangle-shaped vertices has additional impact comparable to that of those large banks, even though no large banks are members of that community.

[11]Recall that we measure financial stress rather than monetary losses. We can conceive financial stress as potential losses that may or may not materialize at all.

Table 3: *Evaluation of the additional and initial shock stress per each community encountered in December 2014 with $\sigma = 1$ million BRL (see Fig. 9a).*

	Stress Measures		Community Composition (Bank Size)			
Community	DebtRank	Initial Shock	Micro	Small	Medium	Large
Circle	0.106	0.129	7	9	3	3
Square	0.017	0.024	9	3	1	0
Diamond	0.024	0.026	5	5	0	0
Triangle	0.031	0.056	4	3	2	0

6 Conclusions

In this paper, we develop systemic risk measures for the Brazilian interbank market by using a novel network-based framework that modulates the impact effects that we obtain from stress measures. The framework relies on a network approach because networks are able to capture topological aspects of the data relationships, which may help in extracting nonlinear features of the risks embodied into the FIs relationships. Moreover, the network representation is able to unify the structure, dynamics, and functions of the system that it represents. It does not only describe the interaction among FIs (structure) and the evolution of such interactions (dynamics), but also reveals how both of them—structure and dynamics—affect the overall function of the network.

Applying our framework for a 1-year horizon, we find that medium-sized banks are the main contributors for the expected systemic risk in the Brazilian interbank market, rather than large-sized banks. The expected systemic stress of large banks is largely reduced by virtue of their low DPs. In contrast, medium-sized banks turn out to have larger expected systemic stress because they have non-negligible default probabilities and also inflict moderate additional stress in the financial system.

We also devise novel heuristics for designing stress tests that are feasible and relevant and thus can support financial surveillance. To address the feasibility property, the method relies on finding bank communities whose members present significant pairwise joint default probabilities. To account for the relevance property, the method only considers pairwise connections in which the default of the pair of FIs produces large potential losses. We apply this methodology in the Brazilian interbank market and find that a large portion of the identified communities is composed of non-large banks. We then select as stress-testing scenarios the joint default of all members of a same community. We find that these bank communities can inflict large additional stress in the financial system. Our procedure suggests that, although size and interconnectedness matters, it is crucial to evaluate the emergence of these bank communities, which may also be a trigger for systemic risk.

These findings are based on a framework in which contagion is driven by direct exposures between banks, and initial shocks are bank defaults. This framework can be extended to consider different types of initial shocks and the same contagion mechanism, for instance, considering shocks originating from banks' exposures to the same assets or counterparty types. Another possibility is to study different contagion channels taking into account that some of them may significantly amplify the initial shocks. These investigations are left for future work, that, after completion, will contribute to a more complete identification of the risks incurred by the banking system under study.

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Appendix A Individual default probabilities

We use the Merton (1974)'s structural model to estimate the DP of individual banks, which models credit risk using the Contingent Claim Analysis[12]. The intuition of this model is to consider the bank's assets as the underlying asset of an European call option, with strike price equal to its obligations and time to maturity T. If the bank defaults, equity holders receive nothing, because the bank does not have enough resources for repaying its obligations. Otherwise, if it does not default, equity holders receive the difference between the values of assets and liabilities. The model also considers that the bank defaults if its assets value falls below its distress barrier (*DB*), which is not necessarily equal to its obligations' value, due to liquidity shortages, contract breaches and other similar problems. We compute the *DB* based on the KMV (2001) model, using accounting data, as:

$$DB = STD + \alpha LTD \tag{20}$$

in which STD and LTD stand for the short- (maturity $\leq$ 1 year) and long-term (maturity above 1 year) liabilities, respectively, and α is a parameter between 0 and 1 that proxies the share of long-term liabilities of a bank subjected to early redemption in case of stress. Following Moody's-KMV[13], we fix $\alpha = 0.5$.

Applying these definitions to the model proposed in Black and Scholes (1973), we compute the option's payoff received by equity holders as:

$$E = \max(A\ \mathcal{N}(d_1) - DB\ e^{-rT}\ \mathcal{N}(d_2), 0) \tag{21}$$

In (21), A is the assets' value, r is the risk-free interest rate, $\mathcal{N}(\cdot)$ is the cumulative normal distribution,

$$d_1 = \frac{\ln(\frac{A}{DB}) + (r + \frac{\sigma_A^2}{2})T}{\sigma_A \sqrt{T}} \tag{22}$$

and

$$d_2 = \frac{\ln(\frac{A}{DB}) + (r - \frac{\sigma_A^2}{2})T}{\sigma_A \sqrt{T}}, \tag{23}$$

in which σ_A denotes the assets volatility.

From Black and Scholes (1973), it is possible to obtain the equation that relates assets and equity volatilities:

[12]Contingent Claim Analysis is a generalization of the option pricing theory presented in Black and Scholes (1973).

[13]According to Souto et al. (2009), Moody's-KMV uses α in the range $0.5 - 0.6$ based on the calibration of their model. This intends to match model and historical probabilities of default.

$$\sigma_E = \mathcal{N}(d_1)\frac{A}{E}\sigma_A. \quad (24)$$

With information on the market value and on the equity volatility, and on the book value of liabilities, it is possible to estimate the implied value for A and σ_A by solving the system of equations displayed in (21) and (24).

The time to maturity T, usually assumed to be 1 year, is the horizon for which we compute DP. We consider that a 1-year time horizon is consistent with i) the usual assets' classification into short-term and long-term liabilities that the model requires, and ii) the time given to banks to adapt to raises in capital.[14] We assume that the bank's asset values are log-normally distributed, which, according to Crouhy et al. (2000), is a robust assumption. We also consider that investors are risk-neutral, that is, they demand as return rate the risk-free return rate r, which is lower than that required by risk-averse investors. This assumption results in conservative (higher) DP estimates. Therefore, the DP of a bank in a time horizon T, computed in $t = 0$, is given by:

$$\begin{aligned} DP &= P(A_T \leq DB) \\ &= P(\ln A_T \leq \ln DB) \\ &= \mathcal{N}\left(-\frac{\ln(\frac{A_0}{DB}) + (r - \frac{\sigma_A^2}{2})T}{\sigma_A\sqrt{T}}\right) \\ &= \mathcal{N}(-d_2), \end{aligned} \quad (25)$$

in which A_T is the banks' total assets at time T. We also compute the distance to distress ($D2D$) for a risk-neutral environment, as:

$$D2D = -d_2, \quad (26)$$

which is the distance of the bank's assets value to the distress barrier in $t = 0$, measured in assets value' standard deviations.

Appendix B Pairwise default probabilities

We compute joint DPs for each pair of banks using the consistent information multivariate density optimizing (CIMDO) methodology (Segoviano (2006)). We first select a prior bivariate distribution that represents our initial belief on the asset values, which we choose on the grounds of theoretical arguments and economic intuition, that

[14] For instance, when capital requirements increase, under the countercyclical capital buffer framework of Basel III, banks have one year to comply (BCBS (2010)).

may not necessarily fully comply with the empirical observations. Then, the main idea of the methodology consists in using Kullback (1959)'s cross-entropy framework to derive a posterior distribution from sucessive adjustments of the selected prior distribution that complies with the empirically observed individual DPs and distances to distress.[15] Finally, we use this posterior bivariate distribution to compute the joint DPs for the pair of banks.

The CIMDO methodology assumes that the premises of the structural approach hold, that is, that a bank's underlying asset value evolves according to a stochastic process and that the bank defaults if its assets value falls below a distress barrier, as exposed in Appendix A. Additionally, it considers that the bank i's logarithmic asset values x_i are normally distributed and standardize these values so that $x_i \sim \mathscr{N}(0,1)$, with $x_i = 0$ in $t = 0$. It also applies this standardization to distances to default $D2D_i$ and distress barriers DB_i. Finally, it considers that the default region is in the upper part of the distribution[16] and that the logarithmic asset values x_i and x_j are correlated according to a time-invariant correlation structure ρ. The CIMDO methodology derives, for the pair of banks i and j, the bivariate distribution function of x_i and x_j from a theoretically-assumed bivariate prior distribution and individual banks data DP_i, DB_i, DP_j and DB_j. According to Segoviano (2006), calibrating the parametric prior distribution using these data does not generally result in an optimal representation of the true underlying distribution function. Thus, he proposes using the Kullback (1959)'s cross-entropy minimization procedure to recover the distribution that is most consistent with the banks' DP and DB constraints, while minimizing the entropic distance of that distribution in relation to the prior distribution. We refer to this process as prior distribution update scheme with empirical data to obtain a posterior distribution.

To compute the bivariate posterior distribution density through a cross-entropy minimization procedure, we solve the following optimization problem:

$$\widehat{p(x_i,x_j)} = \text{argmin}_{p(x_i,x_j)}\, C[p,q] \tag{27}$$

subject to:

$$\int\int p(x_i,x_j)\mathbb{1}_{\{DB_i,\infty\}} dx_i dx_j = DP_i, \tag{28}$$

$$\int\int p(x_i,x_j)\mathbb{1}_{\{DB_j,\infty\}} dx_j dx_i = DP_j, \tag{29}$$

$$\int\int p(x_i,x_j) dx_i dx_j = 1, \tag{30}$$

$$p(x_i,x_j) \geq 0. \tag{31}$$

[15]See Appendix A for a detailed discussion on how to evaluate individual DPs and distances to distress

[16]This happens because after standardization, $D2D_i > 0$ when $DB_i < 0$

in which $C[p,q]$ stands for the entropic distance between the posterior and prior distributions p and q, respectively. We evaluate $C[p,q]$ as follows:

$$C[p,q] = \int\int p(x_i,x_j) \ln\left[\frac{p(x_i,x_j)}{q(x_i,x_j)}\right] dx_i dx_j. \tag{32}$$

In the optimization problem above, Equation (27) is the objective function that minimizes the entropic distance $C[p,q]$, Equations (28) and (29) are restrictions related to marginal default probabilities of banks i and j in $t = 0$, and Equations (30) and (31) ensure that the solution of optimization problem, $\widehat{p(x_i,x_j)}$, is a valid density, i.e., that it adds to 1 over its support and that it satisfies the non-negativity condition. Besides, $\widehat{p(x_i,x_j)}$ is the estimate of the bivariate posterior distribution density being computed and $\mathbb{1}_{\{DB_b,\infty\}}$ is an indicator function that identifies the default region of bank b, considering that the default region is in the upper part of the distribution. For bank b, $\mathbb{1}_{\{DB_b,\infty\}} = 1$ if $x_b \geq DB_b$, being zero otherwise.

Solving the problem using calculus of variations, we obtain the following closed-form equation for the posterior bivariate density:

$$\widehat{p(x_i,x_j)} = q(x_i,x_j)\exp\{-\left[1+\hat{\mu}+\hat{\lambda}_1\mathbb{1}_{\{DB_i,\infty\}}+\hat{\lambda}_2\mathbb{1}_{\{DB_j,\infty\}}\right]\}. \tag{33}$$

In the equation above, $\hat{\lambda}_1$, $\hat{\lambda}_2$ and $\hat{\mu}$ are Lagrange multipliers. To find them, we numerically solve the following 3-equation system:

$$\begin{aligned}
&\int_{-\infty}^{\infty}\int_{DB_i}^{\infty} \widehat{p(x_i,x_j)} dx_i dx_j = DP_i,\\
&\int_{-\infty}^{\infty}\int_{DB_j}^{\infty} \widehat{p(x_i,x_j)} dx_j dx_i = DP_j,\\
&\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \widehat{p(x_i,x_j)} dx_i dx_j = 1,
\end{aligned} \tag{34}$$

in which we use the integrals of (33), the restrictions in (28)–(30) and the integrals computed for the prior $q(x_i,x_j)$ to arrive at the previous system of equations.

The integrals in (34) can be easily computed as we employ indicator functions in (33) that are constant along continuous intervals.

In this paper, we follow the steps below to compute the pairwise default probabilities for banks i and j:

- We compute the empirical data[17] for both banks: DP_i, DB_i, DP_j and DB_j.

- We compute the correlation coefficient ρ of banks i and j log-returns along the previous year using monthly data, following Segoviano and Goodhart (2009) and Guerra et al. (2016).

- Following Guerra et al. (2016), we adopt a bivariate t-distribution with 5 degrees of freedom and correlation ρ as the prior distribution.

- We find the Lagrange multipliers $\hat{\lambda}_1$, $\hat{\lambda}_2$ and $\hat{\mu}$ solving the system of equations in (34).

- We compute the joint default probability DP_{ij}:

$$DP_{ij} = \int_{DB_i}^{\infty} \int_{DB_j}^{\infty} \widehat{p(x_i, x_j)} dx_i dx_j \tag{35}$$

Appendix C Network modularity

The modularity measure quantifies how good a particular division of a network in communities is (Newman and Girvan (2004)). Communities are subgroups of vertices within which the connections are dense but between which they are sparser. Technically, the modularity captures the fraction of edges that falls within communities minus the expected value of the same quantity if we rearrange edges at random, conditional on the given community. Mathematically, we compute modularity in weighted networks as follows (Newman (2004)):

$$Q = \frac{1}{2E} \sum_{i,j \in \mathscr{V}} \left(\mathbf{A}_{ij} - \frac{s_i s_j}{2E} \right) \mathbb{1}_{\{c_i = c_j\}}, \tag{36}$$

in which $\mathscr{V}$ is the set of vertices in a graph; E represents the total edge weight inside the network; s_i stands for the strength (weighted degree) of vertex i; c_i is the community of vertex i; and $\mathbf{A}_{ij}$ is the edge weight between vertices i and j. The indicator function guarantees that we only take into account links inside a community. We evaluate the quantities s_i and E using the following expressions:

[17]Considering that the CIMDO methodology uses DB and $D2D$ data standardized to an assets value distribution $x \sim \mathscr{N}(0,1)$, and that Segoviano (2006) defines the default region as being in the upper part of the distribution, we have that $DB = -D2D$. To associate a default region in the upper part of the distribution with a positive $D2D$, we simply take the opposite sign of the DB value. That is possible because we use a symmetric prior.

$$s_i = \sum_{j \in \mathscr{V}} \mathbf{A}_{ij}, \tag{37}$$

$$E = \frac{1}{2} \sum_{i,j \in \mathscr{V}} \mathbf{A}_{ij} = \frac{1}{2} \sum_{i \in \mathscr{V}} s_i. \tag{38}$$

Inside the summation in (36), we remove the edge weights that are expected to occur due to randomness, using a random network model. Therefore, the modularity effectively reflects the excess of concentration of links within modules in relation to a random distribution of links.

The modularity is a global network measure that ranges from 0 to 1. When the modularity is near 0, it means that the network does not present community structure, suggesting that the links are disposed at random in the network. As the modularity grows, the community structure gets more and more defined, that is, the mixture between communities gets smaller and therefore the fraction of links inside communities is larger than that between different communities. Empirically, modularity values larger than 0.3 already indicate strong evidence of well-defined communities.