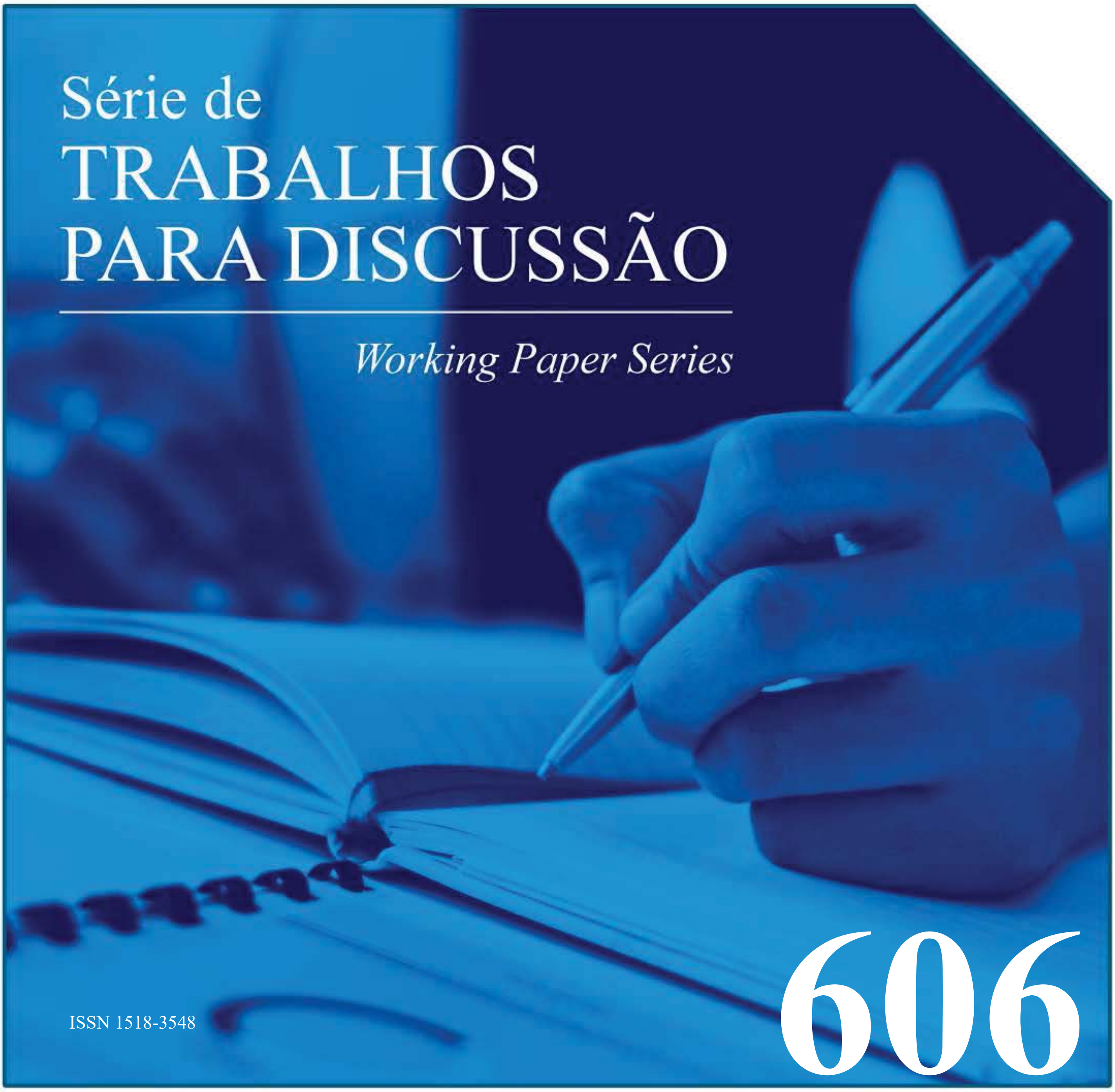

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Smoothing the New-Keynesian Capital Puzzle

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Non-Technical Summary

This paper investigates the mechanics of New-Keynesian economic models, focusing on the role of capital and interest rates in the transmission of monetary policy. A key concern addressed is the real interest rate channel of monetary policy transmission, which describes how changes in a central bank's interest rate influence inflation and economic activity through their effect on the inflation-adjusted expected return on investments.

Recent research by Rupert and Šustek (2019) suggests that including endogenous capital (determined within the model) in these models allows real interest rates to react in various ways following a monetary shock, challenging the established understanding that real interest rates increase with monetary tightening. This inconsistency, they argue, makes it difficult to rely on these models for accurate policy recommendations.

The paper responds to this challenge by incorporating interest-rate smoothing — central bank practice of adjusting interest rates gradually rather than abruptly — into the models. This addition helps align the models' predictions with real-world observations that real interest rates typically increase after a positive monetary shock, reaffirming the validity of the real interest rate channel in policy analysis.

The findings suggest that, while the initial introduction of endogenous capital complicates the prediction of real interest rate movements, adding interest-rate smoothing considerably resolves this issue. Such an adjustment not only maintains the theoretical consistency of New-Keynesian models but also enhances their practical applicability for central banks' policy decisions. The study concludes that, with the appropriate adjustment, New-Keynesian models remain robust tools for understanding and guiding monetary policy.

Sumário Não Técnico

Este artigo investiga a mecânica dos modelos econômicos Novo-Keynesianos, com foco no papel do capital e das taxas de juros na transmissão da política monetária. Uma preocupação central abordada é o canal da taxa de juros real na transmissão da política monetária, que descreve como mudanças nas taxas de juros de um banco central influenciam a inflação e a atividade econômica por meio do seu efeito sobre o retorno esperado dos investimentos ajustado pela inflação do período.

Pesquisa recente de Rupert e Šustek (2019) sugere que incluir capital endógeno (determinado dentro do modelo) nesses modelos permite que as taxas de juros reais reajam de diversas maneiras após um choque monetário, desafiando a compreensão estabelecida de que as taxas de juros reais aumentam com o aperto monetário. Eles argumentam que essa inconsistência dificulta a confiança nesses modelos para recomendações de política precisas.

O artigo responde a esse desafio incorporando suavização da taxa de juros — uma prática em que os bancos centrais ajustam as taxas de juros gradualmente, em vez de abruptamente — nos modelos. Essa adição ajuda a alinhar as predições dos modelos com as observações do mundo real, onde as taxas de juros reais geralmente aumentam após um choque monetário positivo, reafirmando a validade do canal da taxa de juros real na análise de políticas.

Os resultados sugerem que, embora a introdução de capital endógeno complique a predição dos movimentos das taxas de juros reais, a adição de suavização da taxa de juros resolve em grande parte esse problema. Tal ajuste não só mantém a consistência teórica dos modelos Novo-Keynesianos, mas também melhora sua aplicabilidade prática para as decisões de política dos bancos centrais. O estudo conclui que, com o ajuste apropriado, os modelos Novo-Keynesianos continuam sendo ferramentas robustas para entender e orientar a política monetária.

Smoothing the New-Keynesian Capital Puzzle

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November 15, 2024

Abstract

Rupert and Šustek (2019) showed that introducing endogenous capital into the canonical New-Keynesian model allows real interest rates to move in any direction after a positive monetary shock. According to them, this would prove that the real interest rate channel of monetary policy transmission is only observational — not structural — in that class of models, and therefore subject to the Lucas (1976) critique. In this paper, I show that such an identification problem for dynamic stochastic general equilibrium (DSGE) and vector autoregression (VAR) models can be circumvented by incorporating interest-rate smoothing — a feature as prevalent in medium-scale New-Keynesian models as capital itself — into the Taylor rule. I find that the negative association between changes in inflation and changes in the real interest rate is actually more robust than that between the former and changes in the nominal interest rate.

Keywords: Monetary Policy, New-Keynesian Model, Interest-Rate Smoothing

JEL Classification: E43; E52; E58.

The views expressed in this paper are those of the author and do not necessarily reflect those of the BCB or the BIS.

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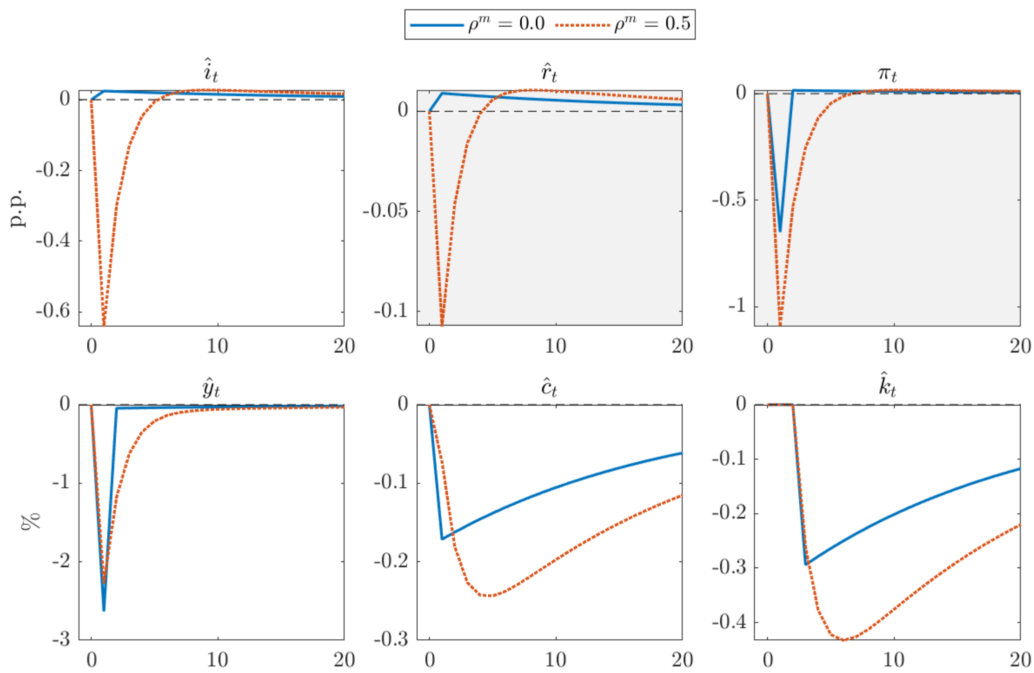
1 Introduction

In a recent paper, Rupert and Šustek (2019) challenged the existence of a real interest rate channel of monetary policy transmission in textbook New-Keynesian models, e.g., Woodford (2003*a*) and Galí (2015). They showed that introducing endogenous capital into such models allows the real interest rate to move in any direction after a positive monetary shock.[1] To illustrate this, Figure 1 displays the effect of a positive monetary shock under two different specifications for its persistence coefficient, ρ^m, a policy parameter. The real interest rate rises immediately after a transitory shock ($\rho^m = 0.0$) but falls when the latter is just mildly persistent ($\rho^m = 0.5$).



Note: hat variables are deviations from the zero-inflation-target steady state. Nominal interest rate ($\hat{i}_t$), real interest rate ($\hat{r}_t$), inflation (π_t), output ($\hat{y}_t$), consumption ($\hat{c}_t$), capital at the beginning of period ($\hat{k}_t$).

Figure 1: Impulse response function to a positive monetary shock in a canonical New-Keynesian model augmented with endogenous capital

According to Rupert and Šustek (2019), this would prove that the real interest rate channel

[1] Woodford (2003*a*, sec. 5.3.3) calls the lack of any effect of variations in private spending on the economy's productive capacity one of the more obvious omissions in the baseline New-Keynesian model. He observes that although there are calibrations for which introducing endogenous capital results in similar dynamics for output and inflation after a monetary shock, the mechanisms within each model that generate these results are not too closely parallel.

is only observational — not structural — in that class of models, a result not robust to the Lucas (1976) critique, raising serious concerns about the use of these models for policy recommendations. For example, the very interpretation of their impulse response functions becomes debatable, and it would be quite problematic to assume such a channel for the identification of vector autoregression (VAR) models, whether through sign restrictions, as in the method proposed by Uhlig (2005); through sequentially ordering nominal and real rates, as in a Cholesky decomposition (Benoit (1924)); or by selecting real rates instead of nominal ones as part of the model's endogenous variables. In this paper, I show that this identification problem, which is a puzzle for the New-Keynesian theory, can be largely circumvented in the relevant parameter range by adding interest-rate smoothing — a feature as prevalent as capital in medium-scale New-Keynesian dynamic stochastic general equilibrium (DSGE) models, e.g., Smets and Wouters (2003, 2007) — to the Taylor rule.

The importance of Rupert and Šustek (2019)'s result is straightforward yet significant to the extent that it has recently been evoked by Holden (2024) as one of the reasons to motivate a radical shift in central banking from nominal to real interest rate rules. Although in general equilibrium all variables are determined simultaneously, every model needs a story to tell, and the common view on the transmission of monetary policy in textbook New-Keynesian models relies on the real interest rate channel, which Ireland (2010) describes as follows:

> "A monetary tightening in the form of a shock to the Taylor rule that increases the short-term nominal interest rate translates into an increase in the real interest rate as well when nominal prices move sluggishly due to costly or staggered price setting. This rise in the real interest rate then causes households to cut back on their spending, as summarized by the IS curve. Finally, through the Phillips curve, the decline in output puts downward pressure on inflation, which adjusts only gradually after the shock."

Galí (2015, p. 5) also emphasizes the real interest rate channel when describing the short-run non-neutrality of monetary policy in this class of models:

> "As a consequence of the presence of nominal rigidities, changes in short-term nominal interest rates (whether chosen directly by the central bank or induced by changes in the money supply) are not matched by one-for-one changes in expected inflation, thus leading to variations in real interest rates. The latter

bring about changes in consumption and investment and, as a result, in output and employment, because firms find it optimal to adjust the quantity of goods supplied to the new level of demand. In the long run, however, all prices and wages adjust, and the economy reverts back to its natural equilibrium."

Schematically, a positive monetary shock (ε_t^m) should increase the real interest rate (R_t), *because of sticky prices*, leading to the reduction of consumption (C_t), output (Y_t), and, finally, inflation (Π_t).

$$\uparrow \varepsilon_t^m \quad \Rightarrow \quad \underbrace{\uparrow R_t}_{\text{if prices are sticky}} \quad \Rightarrow \quad \downarrow C_t \quad \Rightarrow \quad \downarrow Y_t \quad \Rightarrow \quad \downarrow \Pi_t$$

However, Rupert and Šustek (2019) propose a different story, which they argue is more consistent with the actual mechanics of the model. The transmission does not operate through a real interest rate channel. First, equilibrium inflation is approximately determined as in a flexible-price model.[2] Second, output is pinned down by the New-Keynesian Phillips curve, interpreted here to mean that, given the expected inflation trajectory, firms that cannot adjust prices will change output. Finally, the real rate only reflects the feasibility of keeping consumption smooth when income changes, whereas consistency with the real interest rate channel depends on the persistence of the monetary shock. The canonical model, with fixed capital, is simply a limiting case where the latter adjustment costs are infinite. According to this view, monetary transmission should work as follows:

$$\uparrow \varepsilon_t^m \quad \Rightarrow \quad \downarrow \Pi_t \quad \Rightarrow \quad \underbrace{\downarrow Y_t}_{\text{if prices are sticky}} \quad \Rightarrow \quad \downarrow C_t \quad \Rightarrow \quad \underbrace{?R_t}_{\substack{\text{depends on the presence} \\ \text{of capital and calibration}}}$$

I adopt the following modeling strategy to challenge the practical relevance of the aforementioned finding. First, I solve the textbook New-Keynesian model of Galí (2015) with capital and interest-rate smoothing in the Taylor rule to show that the latter can circumvent the identification problem revealed by Rupert and Šustek (2019). This finding that the real

[2] In Chapter 2 of Galí (2015), a canonical real business cycle (RBC) model is augmented with a fixed-intercept interest rate rule to pin down inflation and, thus, a trajectory for the price level. Current inflation, as a deviation from its steady-state value, is determined by the expected path of real interest rate deviations from the steady state, as long as the Taylor principle is obeyed. It is important to note that the steady-state value of the real interest rate and the intercept of the monetary policy rule coincide, assuming a zero-inflation target. Chapter 1 of Woodford (2003*a*) shows the same idea in a partial-equilibrium monetary model where the sequence of real interest rates is exogenous.

interest rate channel of monetary policy is reestablished with a common ingredient of medium-scale New-Keynesian models weakens the concerns about its correct identification in policy-oriented DSGE or VAR models, where the latter are mostly immune to the problem because lagging terms are ubiquitous in their specification. Then, I better qualify my result by exploring different combinations of interest-rate smoothing and capital adjustment costs. Making capital adjustment sluggish is warranted to prevent output from overreacting after a monetary shock.

Accepting that at least some smoothing is the rule in central banking and that capital adjustment costs are never negligible in the real world leads to an even broader conclusion. Regarding sign identification, the relationship between inflation and real interest rates is actually more robust than that between the former and nominal interest rates. That is because, after a monetary shock, there is no univocal direction for the observed nominal interest rate response, whose rule is composed of both an endogenous (i.e., response to inflation) and an exogenous component (the shock). A sufficiently persistent positive shock can depress inflation expectations to such an extent that current inflation subsides to the point that current nominal interest rate must also decrease. This textbook result has been explored by both Woodford (2003*a*, sec. 4.2.4) and Galí (2015).

The next sections of this paper are structured as follows. Section 2 presents the related literature. Section 3 describes, solves, and analyzes the New-Keynesian model before and after introducing endogenous capital and interest-rate smoothing. Section 4 takes the proposed solution to a medium-scale model. Finally, Section 5 concludes.

2 Related literature

This paper builds on Rupert and Šustek (2019), who scrutinize the mechanics of canonical New-Keynesian models, i.e., those in Woodford (2003*a*) and Galí (2015). They argue that the monetary policy transmission mechanism in this class of models does not operate through the real interest rate channel, contrary to the conventional view. The observational similitude with the real interest rate channel would come from an implicit assumption of infinite capital adjustment costs.

Brault and Khan (2022) modify Rupert and Šustek (2019)'s work to include frictions on changes in the flow of investments rather than on capital adjustment. They find that the real interest rate always moves in the same direction as the monetary policy shock, regardless of adjustment cost size or shock persistence. The authors argue that, at least in contemporary (medium-scale) New-Keynesian models, the real interest rate channel is present, a point similar to mine but made with a different ingredient.

Suspicion about the real interest rate channel of New-Keynesian models is not new. The seminal work of Kimball (1995) on the derivation of a real business cycle model with sticky prices – he called it Neo-Monetarist – dedicates a whole section to discussing the unlikelihood of that channel. He concludes that, even when investment adjustment costs are introduced, parameter values perceived by him as "plausible" would imply that the real interest rate increases in response to a monetary expansion.[3] The "implausible" scenario would occur if either adjustment costs were "too high" or convergence back to the long-run equilibrium after a monetary shock was "too fast", not unlike what Rupert and Šustek (2019) find. Nonetheless, here lie two distinctions between the Neo-Monetarist model and most of the New-Keynesian models that followed. First, while Kimball (1995) insisted on portraying monetary policy through a quantity equation with exogenous shocks to the money supply, the New-Keynesian literature has followed the real-world trend of adopting nominal interest rate rules with an endogenous response to inflation. Especially when augmented with smoothing, as I propose in this paper, these last rules put in sharp relief the speed of the convergence back to a long-run equilibrium. Second, the parameterization he deems as "plausible" – an investment adjustment cost elasticity of 0.2 and a (labor-constant) elasticity of intertemporal substitution (EIS) for consumption of 0.2 – does not match modern estimations of these models, which find higher values for the EIS.[4]

My modification of the canonical model is empirically motivated. The presence of significant interest-rate smoothing in the response function of the Federal Reserve is found by Clarida, Galí and Gertler (1999), for both the pre-Volcker (1960:1-1979:2) and the Volcker-Greenspan (1979:3-1996:4) eras, as well as by Coibion and Gorodnichenko (2012), whose results employing both hard and narrative real-time data favor that source of purposeful policy inertia over serially correlated monetary shocks, either arbitrary or motivated by, perhaps, inherited persistence from the underlying data generating process of omitted vari-

[3]The model is linearized and, therefore, I assume a symmetrical response in the case of a monetary contraction.

[4]Using Bayesian methods, Smets and Wouters (2003) estimate the EIS to be 0.74 for the Euro Area, and Smets and Wouters (2007) estimate it to be 0.68-0.72 for the U.S. All values are posterior modes.

ables to which the Federal Reserve also responds. For their part, Smets and Wouters (2007) estimate a medium-scale New-Keynesian DSGE model using Bayesian methods for the United States and find considerable coefficients for interest-rate smoothing (above 0.7) as well as small coefficients for monetary shock persistence (below 0.3). These papers suggest the empirical presence of smoothing through the estimation of either single or multiple equation models, that is, by imposing only a little or a lot of informational restriction on the estimation.

Nonetheless, contrasting results can still be found depending on the estimation strategy as Rudebusch (2006) and Carrillo, Fève and Matheron (2018) demonstrate, favoring the modeling of serially correlated monetary shocks employed by Rupert and Šustek (2019). Both approaches to monetary policy are not mutually exclusive, though, since data-driven monetary policy and contingency on new information do not preclude mild forms of forward guidance and policy inertia. This paper shows that different combinations of these two features are enough to restore the canonical identification of the real interest rate channel.

My modification is also theoretically justified. Sack and Wieland (2000) and Woodford (2003*b*) show that smoothing policy interest rates may be optimal from a welfare perspective, a concern already presented in Goodfriend (1987) in terms of a central bank's preference to maintain "orderly money markets" by minimizing unexpected asset price movements that otherwise could raise the risk of bankruptcies and banking crises.

Thus, although smoothing is a policy choice, high levels of it are generally optimal, and low levels are empirically rare, which warrants the case of this paper.

3 New-Keynesian model before and after capital

In this section, I propose and solve a New-Keynesian toy model: first the canonical version, then the model augmented with endogenous capital and interest-rate smoothing. This exposition strategy facilitates the comparison.

3.1 Canonical closed economy

Let us consider a closed economy without fiscal policy, where a one-period risk-free nominal bond is available in zero net supply and the central bank adopts a fixed-intercept Taylor rule. I expand here on the simplified presentation made by Rupert and Šustek (2019) of the canonical New-Keynesian model of Galí (2015), with minor notational changes.

The simple model starts with seven variables: real consumption, c_t; labor l_t; real output, y_t; real wage, w_t; real marginal cost, χ_t; nominal interest rate, i_t; and inflation, π_t. Overlined variables represent their steady-state values. There are six parameters: the subjective discount factor, β; the inverse of the elasticity of labor supply, η; the elasticity of substitution between intermediate goods, ε; the fraction of producers not adjusting prices at any given period, θ; the intercept of the Taylor rule, i; and the Taylor-rule coefficient that gauges the central bank's reaction to current inflation, ν. There is also an exogenous monetary shock variable, ξ_t^m.

Assuming a per-period utility function of the form

$$u_t = \log(c_t) - \frac{l_t^{1+\eta}}{1+\eta} \tag{1}$$

and an intermediate goods aggregator like

$$y_t = \left[\int y(j)^{\varepsilon} \, \mathrm{d}j \right]^{\frac{1}{\varepsilon}} \tag{2}$$

the equilibrium conditions of that economy are given by the Euler equation (3) in conjunction with equations (4) to (9), namely the first-order conditions (FOC) of labor, the production function, the real marginal cost, the New-Keynesian Phillips curve under Calvo pricing already linearized around a zero steady-state inflation, a Taylor rule, and the market-clearing condition.

$$\left.\begin{aligned}
\frac{1}{c_t} &= \beta \mathbb{E}_t \left(\frac{1}{c_{t+1}} \frac{1+i_t}{1+\pi_{t+1}} \right) && (3)\\
\frac{w_t}{c_t} &= l_t^{\eta} && (4)\\
y_t &= l_t && (5)\\
\chi_t &= w_t && (6)\\
\pi_t &= \frac{(1-\theta)(1-\theta\beta)}{\theta} \left(\mathrm{mc}_t - \overline{\mathrm{mc}} \right) + \beta \mathbb{E}_t \pi_{t+1} && (7)\\
i_t &= i + \nu \pi_t + \xi_t^m && (8)\\
y_t &= c_t && (9)
\end{aligned}\right\}$$

As usual, the equilibrium conditions above can be simplified to a three-equation system linearized around a nonstochastic steady state ($\overline{\pi} = 0$, $\overline{y} = 1$). This is possible by first linearizing (3) and substituting the market-clearing condition (9) into it, then eliminating (9), (5), and (6) through the substitution of their respective expressions for c_t, l_t, and w_t into (4), which is later linearized such that $\hat{y}_t \equiv \frac{y_t - \overline{y}}{\overline{y}}$. Finally, the Taylor Rule (8) is rewritten as deviations of the interest rate from its steady-state value such that $\hat{i}_t = i_t - i$.

$$\left.\begin{aligned} -\hat{y} &= -\mathbb{E}_t \hat{y}_{t+1} + \hat{i}_t - \mathbb{E}_t \pi_{t+1} \quad (10)\\ \pi_t &= \Omega \hat{y}_t + \beta \mathbb{E}_t \pi_{t+1} \quad (11)\\ \hat{i}_t &= \nu \pi_t + \xi_t^m \quad (12) \end{aligned}\right\}$$

where

$$\Omega \equiv \frac{(1+\eta)(1-\theta)(1-\theta\beta)}{\theta} > 0 \quad (13)$$

Notice that when prices are fully flexible, $\theta \to 0$, then $\Omega \to \infty$, whereas when prices are fixed, $\theta \to 1$, then $\Omega \to 0$. As Rupert and Šustek (2019) comment, it is helpful to think of Ω as a weight that gauges the solution coefficients of the system between the fully flexible and the fixed price regime.

I can proceed further by substituting the policy rule (12) into (10) so that I reduce the system to only two equations:

$$\left.\begin{aligned} -\hat{y} &= -\mathbb{E}_t \hat{y}_{t+1} + \nu \pi_t + \xi_t^m - \mathbb{E}_t \pi_{t+1} \quad (14)\\ \pi_t &= \Omega \hat{y}_t + \beta \mathbb{E}_t \pi_{t+1} \quad (15) \end{aligned}\right\}$$

I assume the monetary shock follows an AR(1) process given by $\xi_t^m = \rho^m \xi_{t-1}^m + \epsilon_t^m$, where $\rho^m \in [0,1)$ and ϵ_t^m is i.i.d. N(0,1). Solving the model with the method of undetermined coefficients — also known as guess-and-verify — by conjecturing $\hat{y}_t = a\xi_t^m$ and $\pi_t = b\xi_t^m$, where a and b are the coefficients I want to obtain, and discarding explosive paths for output and inflation leads to

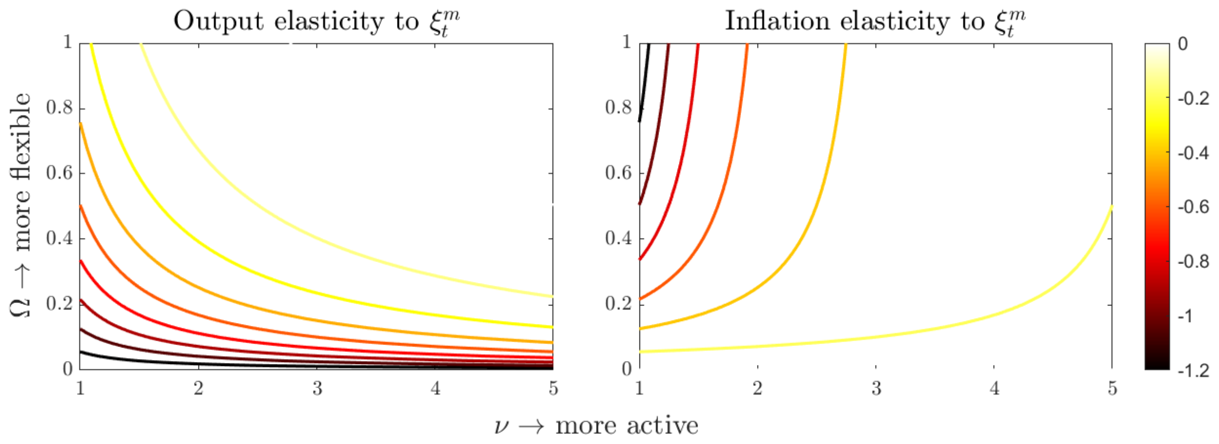
$$a = -\frac{1-\beta\rho^m}{(1-\rho^m)(1-\beta\rho^m) + \Omega(\nu - \rho^m)} < 0 \quad (16)$$

$$b = -\frac{1}{(1-\rho^m)(1-\beta\rho^m)\Omega^{-1} + (\nu - \rho^m)} < 0 \quad (17)$$

where both coefficients imply that a positive monetary shock always reduces inflation and output in the canonical New-Keynesian model.

Figure 2 plots coefficients a and b for different values of ν and Ω, under the calibration of Rupert and Šustek (2019)[5]. As expected, flexible prices reduce output elasticity to zero at the same time that inflation elasticity is at its maximum. Moreover, a more active monetary policy reduces both elasticities.

[5]The following calibration includes parameters that will be incorporated into the model later in this paper: $\beta = 0.99$, $\eta = 1$, $\varepsilon = 0.83$, $\theta = 0.7$, $\nu = 1.5$, $\rho^m = 0.5$, $\alpha = 0.3$, $\delta = 0.025$.



Note: Darker colors imply higher (absolute) elasticity values.

Figure 2: Output and inflation elasticity to a monetary shock

The real interest rate as a deviation from its steady-state value can be obtained from the Fisher identity, $\hat{R}_t = \hat{i}_t - \mathbb{E}_t \pi_{t+1}$. Substituting my solution, I have

$$\hat{R}_t = \underbrace{\left(1 - \underbrace{\frac{1}{1 + \frac{1-\rho^m}{\nu-\rho^m}\frac{1-\beta\rho^m}{\Omega}}}_{\in[0,1]}\right)}_{\geq 0} \xi_t^m \tag{18}$$

which implies that the real interest rate always increases/decreases right after a positive/negative monetary shock, consistent with the presence of a real interest rate channel of monetary policy transmission.

3.2 Endogenous capital and interest-rate smoothing

I now incorporate endogenous capital and show that adopting interest-rate smoothing in the Taylor Rule can deliver impulse-response functions with the sign consistent with the real interest rate channel in the empirically relevant parameter range. This finding largely minimizes the identification problem from an empirical perspective and provides new insight into its mechanics.

I build on the model of Rupert and Šustek (2019), which assumes there is an economy-wide rental market of capital so that firms can rent capital in every period. In that sense, capital is not firm-specific.[6] Moreover, they assume that whenever households change their

[6] Altig et al. (2011) estimate a New-Keynesian DSGE model for the U.S. and find that this modeling choice

stock of capital, there is a quadratic adjustment cost, $-\frac{\kappa}{2}(k_{t+1}-k_t)^2$, where k_t is the stock of capital inherited from the previous period and $\kappa \geq 0$ is a parameter that governs the size of the adjustment cost in terms of foregone real income.

Resuming from the canonical model of Section 3.1, there is a new Euler equation for the capital asset (19), where $\delta \in (0,1)$ is a depreciation rate, and q_t is the price of capital in terms of current consumption, Tobin's q, such that $q_t \equiv 1+\kappa(k_{t+1}-k_t)$. The production function (5) is replaced by (20), which incorporates capital and labor proportionate to constant returns to scale, where α is the Cobb-Douglas coefficient of capital. Equation (21) is the condition for the optimal mix of capital and labor in production, which comes from the FOC of the firm. The marginal cost (6) is updated to include the rent on capital (22). The resources constraint (9) must now account for the investment flow so markets can clear (23). Finally, I substitute the previous Taylor rule (8), also adopted by Rupert and Šustek (2019), with one that includes interest-rate smoothing (24), whose persistence is governed by $\rho^i \in [0,1)$.

$$\left.\begin{aligned}
\frac{1}{c_t} &= \beta \mathbb{E}_t\left(\frac{1}{c_{t+1}}\left(\frac{r_{t+1}-\delta}{q_t}+\frac{q_{t+1}}{q_t}\right)\right) && (19)\\
y_t &= k_t^{\alpha} l_t^{1-\alpha} && (20)\\
\frac{w_t}{r_t} &= \frac{1-\alpha}{\alpha}\left(\frac{k_t}{l_t}\right) && (21)\\
\chi_t &= \left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} && (22)\\
y_t &= c_t + k_{t+1} - (1-\delta)k_t + \frac{\kappa}{2}(k_{t+1}-k_t)^2 && (23)\\
i_t &= \rho^i i_{t-1} + \left(1-\rho^i\right)(i+\nu\pi_t) + \xi_t^m && (24)
\end{aligned}\right\}$$

After substituting equation (21) into (22) by eliminating r_t, and substituting equation (4) into (20) so as to eliminate l_t, the model is log-linearized around the zero-inflation non-stochastic steady state (Appendix A). For any variable X, $\hat{X} \equiv \frac{X_t-\overline{X}}{\overline{X}}$, with the exception of $\hat{i}_t \equiv i_t - \overline{i}$ and $\hat{r}_t \equiv r_t - \overline{r}$. After that, it is possible to eliminate $\hat{r}_t$, $\hat{\chi}_t$, $\hat{w}_t$, i_t and $\hat{l}_t$ to obtain

for introducing endogenous capital results in firms enduring long spells before readjusting prices, up to 9 quarters on average. They show that firm-specific capital can align that spell more with empirical evidence from micro data to, say, once a year.

the following reduced system with only four equations.

$$
\left.
\begin{aligned}
-\hat{c}_t &= -\mathbb{E}_t\,\hat{c}_{t+1} + \rho^i\,\hat{i}_{t-1} + \left(1-\rho^i\right)\nu\pi_t - \mathbb{E}_t\,\pi_{t+1} + \xi_t^m && (25)\\
-\hat{c}_t &= -\mathbb{E}_t\,\hat{c}_{t+1} + \mathbb{E}_t\,\hat{g}_{t+1} + \overline{r}\,\mathbb{E}_t\left(\hat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\hat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\hat{k}_{t+1}\right) && (26)\\
\pi_t &= \Psi\left(\frac{\eta+\alpha}{1-\alpha}\hat{y}_t - \alpha\frac{1+\eta}{1-\alpha}\hat{k}_t + \hat{c}_t\right) + \beta\,\mathbb{E}_t\,\pi_{t+1} && (27)\\
\hat{y}_t &= \frac{\overline{c}}{\overline{y}}\hat{c}_t + \frac{\overline{k}}{\overline{y}}\hat{k}_{t+1} - (1-\delta)\frac{\overline{k}}{\overline{y}}\hat{k}_t && (28)
\end{aligned}
\right\}
$$

where $\Psi \equiv \overline{\chi}\frac{(1-\theta)(1-\theta\beta)}{\theta}$, such that when prices are flexible, $\Psi \to \infty$. Moreover, $G_{t+1} \equiv \frac{q_{t+1}}{q_t}$ is the capital gain, so $\hat{g}_t = \hat{q}_t - \hat{q}_{t-1} = \overline{\kappa}\left(\hat{k}_{t+1} - \hat{k}_t\right) - \overline{\kappa}\left(\hat{k}_t - \hat{k}_{t-1}\right)$, where $\overline{\kappa} = \kappa\overline{k}$.

To check whether the negative response of real interest rates to a positive monetary shock remains an identification problem, I sweep the combinations of parameter values for $\rho^m \in [0:0.1:0.9, 0.95, 0.99]$ and $\rho^i \in [0:0.1:0.9, 0.95, 0.99]$. Table 1 displays the sign of the reaction of the real interest rate right after the shock for all combinations under $\delta = 0.025$ and $\kappa = 0.0$. Tables 2 and 3 increase κ to 0.1 and 0.5, respectively. As one can see, under the hypothesis of no adjustment costs, ρ^i must be at least 0.95 to guarantee a positive response under all values of ρ^m. However, even a small adjustment cost, like $\kappa = 0.1$, is enough to largely increase the parameter range consistent with a real interest rate channel of monetary policy.

Table 1: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.0$

	$\rho^i = 0$	$\rho^i = 0.1$	$\rho^i = 0.2$	$\rho^i = 0.3$	$\rho^i = 0.4$	$\rho^i = 0.5$	$\rho^i = 0.6$	$\rho^i = 0.7$	$\rho^i = 0.8$	$\rho^i = 0.9$	$\rho^i =0.95$	$\rho^i =0.99$
$\rho^m = 0$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.1$	-	-	-	-	-	-	-	+	+	+	+	+
$\rho^m = 0.2$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m = 0.3$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m = 0.4$	-	-	-	-	-	-	-	-	-	-	+	+
$\rho^m = 0.5$	-	-	-	-	-	-	-	-	-	-	+	+
$\rho^m = 0.6$	-	-	-	-	-	-	-	-	-	-	+	+
$\rho^m = 0.7$	-	-	-	-	-	-	-	-	-	-	+	+
$\rho^m = 0.8$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m = 0.9$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m =0.95$	-	-	-	-	-	-	-	-	+	+	+	+
$\rho^m =0.99$	-	-	-	-	-	+	+	+	+	+	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases.

Table 2: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.1$

	$\rho^i = 0$	$\rho^i = 0.1$	$\rho^i = 0.2$	$\rho^i = 0.3$	$\rho^i = 0.4$	$\rho^i = 0.5$	$\rho^i = 0.6$	$\rho^i = 0.7$	$\rho^i = 0.8$	$\rho^i = 0.9$	$\rho^i = 0.95$	$\rho^i = 0.99$
$\rho^m = 0$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.1$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.2$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.3$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.4$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.5$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.6$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.7$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.8$	-	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.9$	-	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.95$	-	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.99$	-	+	+	+	+	+	+	+	+	+	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases.

Table 3: Parameter sweep with $\delta = 0.025$ and $\kappa = 0.5$

	$\rho^i = 0$	$\rho^i = 0.1$	$\rho^i = 0.2$	$\rho^i = 0.3$	$\rho^i = 0.4$	$\rho^i = 0.5$	$\rho^i = 0.6$	$\rho^i = 0.7$	$\rho^i = 0.8$	$\rho^i = 0.9$	$\rho^i = 0.95$	$\rho^i = 0.99$
$\rho^m = 0$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.1$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.2$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.3$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.4$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.5$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.6$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.7$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.8$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.9$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.95$	-	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.99$	-	+	+	+	+	+	+	+	+	+	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases.

So, how restricting is the direction-switching behavior of the real interest rate in response to a monetary shock for the estimation of VARs and DSGEs? We have seen that, in the presence of interest-rate smoothing, an empirically validated (Coibion and Gorodnichenko, 2012), theoretically desirable (Woodford (2003*b*), Sack and Wieland (2000)), and prevalent feature of medium-scale DSGE models (Smets and Wouters (2003) estimates $\rho^i = 0.956$ for the Euro Area; Smets and Wouters (2007) estimates $\rho^i = 0.75 - 0.84$ for the United States), a plausibly small adjustment cost is enough to reestablish the sign consistency with the real interest rate channel.

3.3 The mechanics

Now, using the method of undetermined coefficients, I make explicit the solution for the real interest rate and compare it to the exposition of Rupert and Šustek (2019).

Originally, there are three state variables ($\hat{k}_t$, ξ^m_{t-1}, $\hat{i}_{t-1}$) and one shock (ϵ^m_t).[7] To reduce the number of coefficients I have to solve for, this representation can be simplified to just three state variables ($\hat{k}_t$, ξ^m_t, $\hat{i}_{t-1}$) using the monetary shock process equation. For the four jump variables, I assume $\hat{c}_t = a_0\hat{k}_t + a_1\xi^m_t + a_2\hat{i}_{t-1}$; $\pi_t = b_0\hat{k}_t + b_1\xi^m_t + b_2\hat{i}_{t-1}$; $\hat{y}_t = d_0\hat{k}_t + d_1\xi^m_t + d_2\hat{i}_{t-1}$; $\hat{k}_{t+1} = f_0\hat{k}_t + f_1\xi^m_t + f_2\hat{i}_{t-1}$. The set of coefficients to be determined for the solution of the full system is $\{a_0, a_1, a_2, b_0, b_1, b_2, d_0, d_1, d_2, f_0, f_1, f_2\}$.

With the log-linearized Fisher relation, $\hat{R}_t = \hat{i}_t - \mathbb{E}_t\pi_{t+1}$, and the Euler equation (25) I can write:

$$
\begin{aligned}
\hat{R}_t &= \mathbb{E}_t\,\hat{c}_{t+1} - \hat{c}_t \\
&= \underbrace{\left(a_0 f_0 - a_0 + a_2\left(1-\rho^i\right)\nu b_0\right)\hat{k}_t + \left(a_0 f_2 - a_2 + a_2\rho^i + a_2\left(1-\rho^i\right)\nu b_2\right)\hat{i}_{t-1}}_{=0 \text{ at the shock}} \\
&\quad + \left(\underbrace{\underbrace{\rho^m a_1 - a_1}_{\text{ex-smoothing}} + \underbrace{a_2\left(1-\rho^i\right)\nu b_1 + a_2}_{\text{smoothing}}}_{\text{indirect effect of capital}} \quad \underbrace{+a_0 f_1}_{\text{direct effect of capital}}\right)\xi^m_t
\end{aligned}
$$

where $\hat{R}$ is the log deviation of the gross real interest rate. When I remove interest-rate smoothing, that is, when $\rho^i = 0$, $a_2 = 0$, $b_2 = 0$, $d_2 = 0$, and $f_2 = 0$, the model is the same as the one portrayed in Rupert and Šustek (2019).

The numerical coefficients can be extracted from the decision rules obtained for first-order solutions in Dynare (Adjemian et al. (2024)). Each coefficient is also a partial derivative with respect to a state variable or a shock (i.e., $a_0 \equiv \frac{\partial \hat{c}_t}{\partial \hat{k}_t}$). With that in mind, I can decompose

[7] In Dynare code, there is an additional state variable, k_{t-1}, that is used just for plotting capital at the beginning of the period.

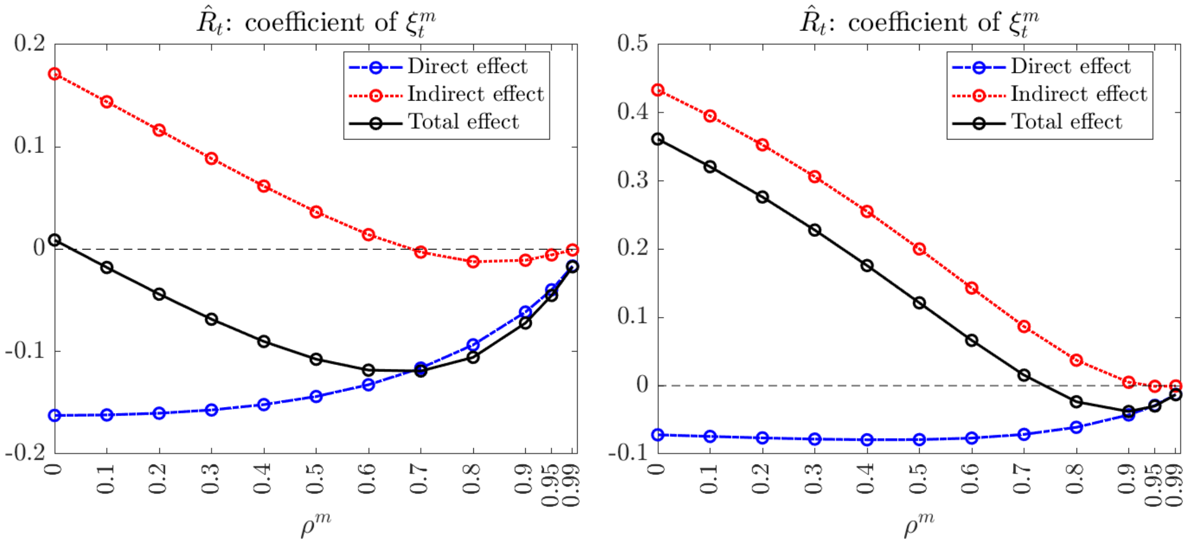
the immediate effect of the shock on the real interest rate into a direct effect of capital and an indirect one. The direct effect is analytically the same as in Rupert and Šustek (2019) since it depends only on the existence of endogenous capital in the model. The indirect effect, on the other hand, can be decomposed into two components: ex-smoothing and smoothing. The ex-smoothing component is the full indirect effect in Rupert and Šustek (2019), whereas the smoothing component appears in my model whenever $\rho^i > 0$. Although the direct effect of capital is always negative, the indirect effect can switch signs depending on how much consumption smoothing is allowed. For that, the shock's persistence, the policy rate's smoothing, and capital adjustment costs are key.[8] I call the total effect the sum of the direct and indirect effects.

$$\frac{\partial \hat{R}_t}{\partial \xi_t^m} = \underbrace{\underbrace{\left(\rho^m - 1\right)\frac{\partial \hat{c}_t}{\partial \xi_t^m}}_{\text{ex-smoothing}} + \underbrace{\left(1 + \left(1-\rho^i\right)\nu \frac{\partial \hat{\pi}_t}{\partial \xi_t^m}\right)\frac{\partial \hat{c}_t}{\partial i_{t-1}}}_{\text{smoothing}}}_{\text{indirect effect of capital}} \quad \underbrace{+\frac{\partial \hat{c}_t}{\partial \hat{k}_t}\frac{\partial \hat{k}_{t+1}}{\partial \xi_t^m}}_{\text{direct effect of capital} < 0}$$

Under the benchmark calibration, with no adjustment costs and no interest-rate smoothing, the direct effect of capital on the real interest rate from a monetary shock is negative for all possible values of ρ^m, while the indirect effect is mostly positive. The absolute indirect effect is larger than the direct one only at the lowest range of ρ^m, as seen in Figure 3a. Note that for considerably persistent monetary shocks ($\rho^m > 0.7$), the indirect effect can be negative, which implies $\frac{\partial \hat{c}_t}{\partial \xi_t^m} > 0$, an atypical situation in which the prospect of a long spell of deflation motivates a consumption increase in the present due to income effect.[9] Meanwhile, in Figure 3b, I show that raising κ to 0.1 increases both components of the total effect, amplifying the range consistent with the real interest rate channel.

[8] The depreciation rate of capital, δ, is also important because it sets the $\overline{k} = \frac{\overline{K}}{\overline{Y}}$, but I prefer to keep it fixed to simplify the analysis.

[9] This case is explored in more detail in Section 3.4.3.



(a) $\kappa = 0$

(b) $\kappa = 0.1$

Figure 3: Decomposition of the effect of capital on $\hat{R}_t$ from a monetary shock when $\rho^i = 0$

In Figure 4, I introduce interest-rate smoothing by setting $\rho^i = 0.5$, with no capital adjustment costs. In that case, the total effect curve becomes flatter near the zero axis. Raising the adjustment cost to $\kappa = 0.1$, as in Figure 5, is enough to turn the total effect curve positive for all possible values of ρ^m even with just a little interest-rate smoothing ($\rho^i = 0.1$).

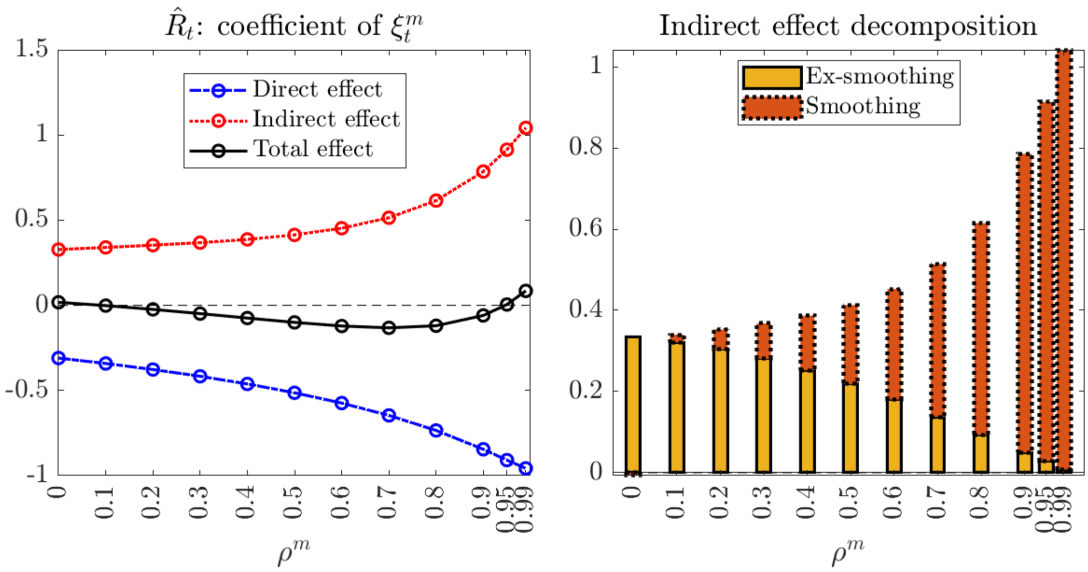


Figure 4: Decomposition of the effect of capital on $\hat{R}_t$ from a monetary shock when $\rho^i = 0.5$ and $\kappa = 0$

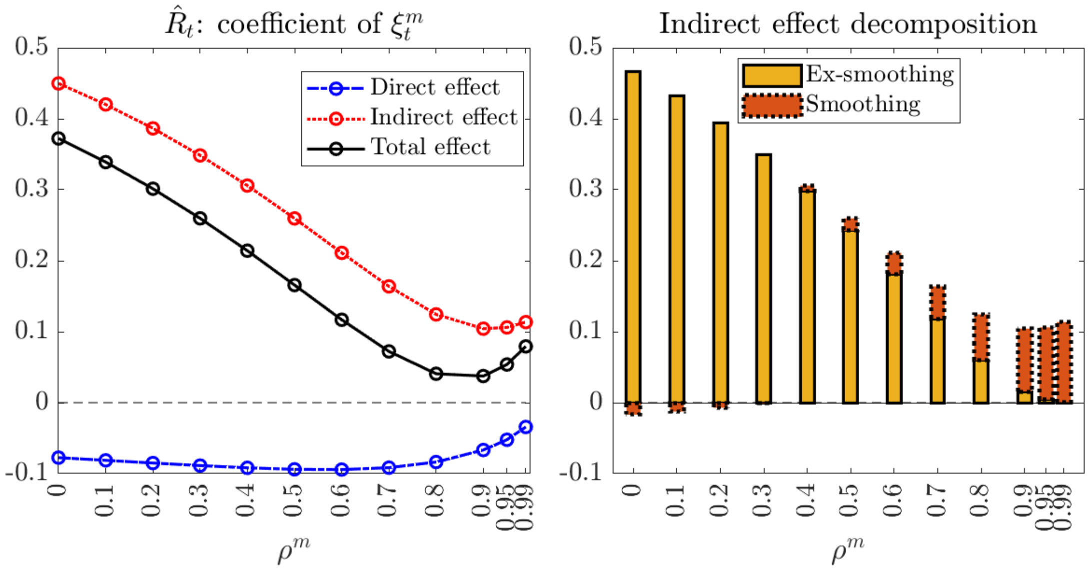


Figure 5: Decomposition of the effect of capital on $\hat{R}_t$ from a monetary shock when $\rho^i = 0.1$ and $\kappa = 0.1$

3.4 Impulse response functions

Next, I plot the impulse response functions of the New-Keynesian model augmented with endogenous capital, adjustment costs, and interest-rate smoothing. I calibrate the standard deviation of the monetary shock to 1 p.p. The graphs display percentage deviations from steady-state values, except for interest rates, which are measured in p.p. deviations from steady-state values. As expected, output, consumption, and inflation respond negatively in the event of a contractionary shock, except for the atypical case in which the income effect dominates the intertemporal substitution of consumption. The capital stock also decreases, but with a lag due to my timing convention. However, the nominal interest rate may react either positively or negatively, as the sign depends on inflation expectations and actual inflation, both of which may decrease significantly in the presence of persistence of the monetary shock, a well-documented pattern (Galí (2015) and Woodford (2003*a*, sec. 4.2.4)).

3.4.1 Fixing with very high interest-rate smoothing

Figure 6 shows that a very high level of interest-rate smoothing ($\rho^i = 0.95$) reestablishes the observational consistency with the real interest rate channel of monetary policy transmission. However, without capital adjustment costs, output overreacts, becoming unrealistic.

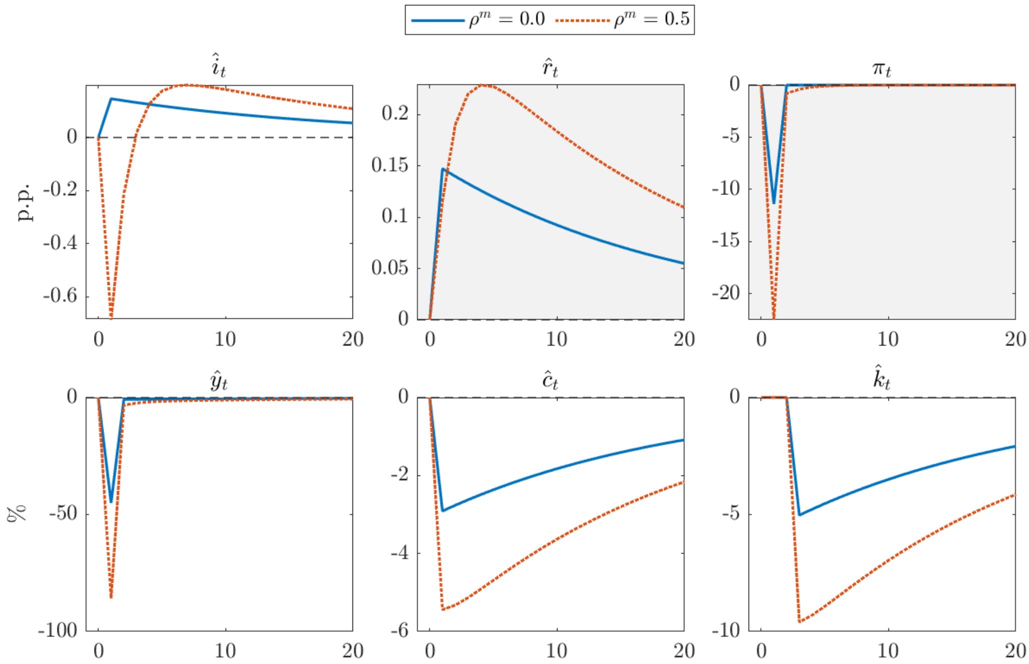


Figure 6: Impulse response function to a one-standard-deviation monetary shock under $\rho^i = 0.95$ and $\kappa = 0$

3.4.2 Fixing with very low interest-rate smoothing and small adjustment cost

Figure 7 shows that simply combining a very low level of smoothing ($\rho^i = 0.1$) with a small adjustment cost ($\kappa = 0.1$) resolves the identification problem. The adjustment cost still prevents output from overreacting right after the shock. Moreover, the negative association between changes in inflation and changes in the real interest rate does not depend on inflation expectations, differing from what is observed for changes in the nominal interest rate, whose sign depends on the persistence of the monetary shock.

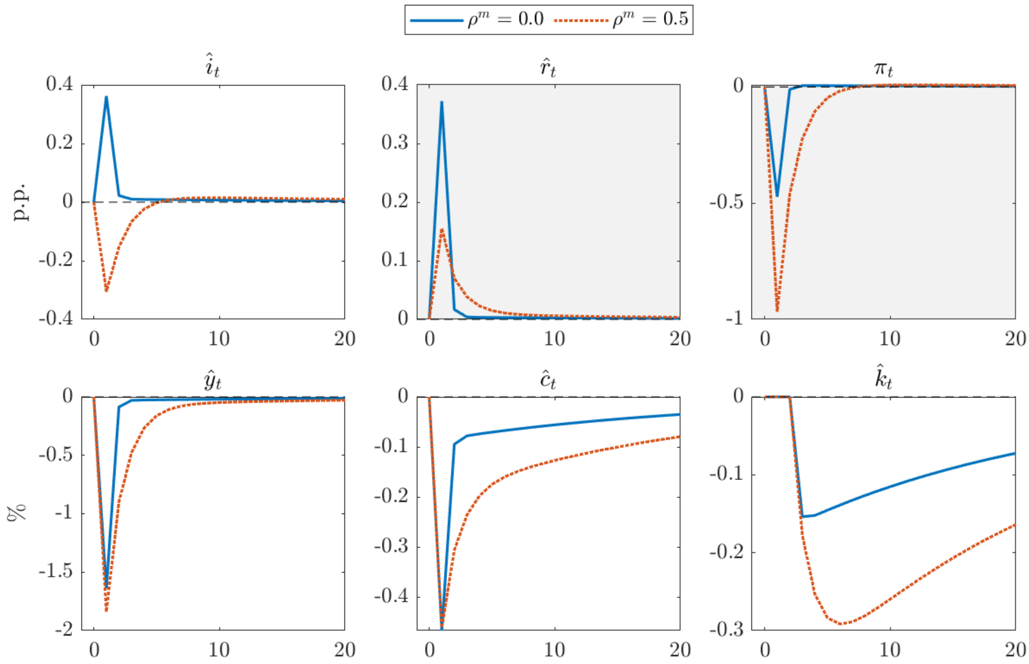


Figure 7: Impulse response function to a one-standard-deviation monetary shock under $\rho^i = 0.1$ and $\kappa = 0.1$

3.4.3 The atypical consumption situation

In Figure 3a, I showed that for considerably persistent monetary shocks, say $\rho^m > 0.7$, the indirect effect of capital can be negative, which implies $\frac{\partial \hat{c}_t}{\partial \xi_t^m} > 0$. This atypical situation in representative agent New-Keynesian (RANK) models appears because the prospect of a long spell of deflation motivates a consumption increase in the present due to investment being much more elastic than output to a monetary shock in the absence of either capital or investment adjustment costs – an extreme case in which the income effect of the shock dominates its intertemporal substitution effect.[10] As can be seen in Figure 8, by comparing the impulse responses to a contractionary monetary shock for $\rho^m = 0.5$ and $\rho^m = 0.95$, the inconsistency with the real interest rate channel is present in this atypical case, but can also be solved with interest-rate smoothing as shown in Figures 4 and 5.

[10] Kaplan, Moll and Violante (2018) point out that, for any reasonable parameterization, monetary policy in RANK models works almost exclusively through intertemporal substitution, in contrast with heterogeneous agent models, in which indirect effects, such as the ones that arise from changes in labor income, play a larger role.

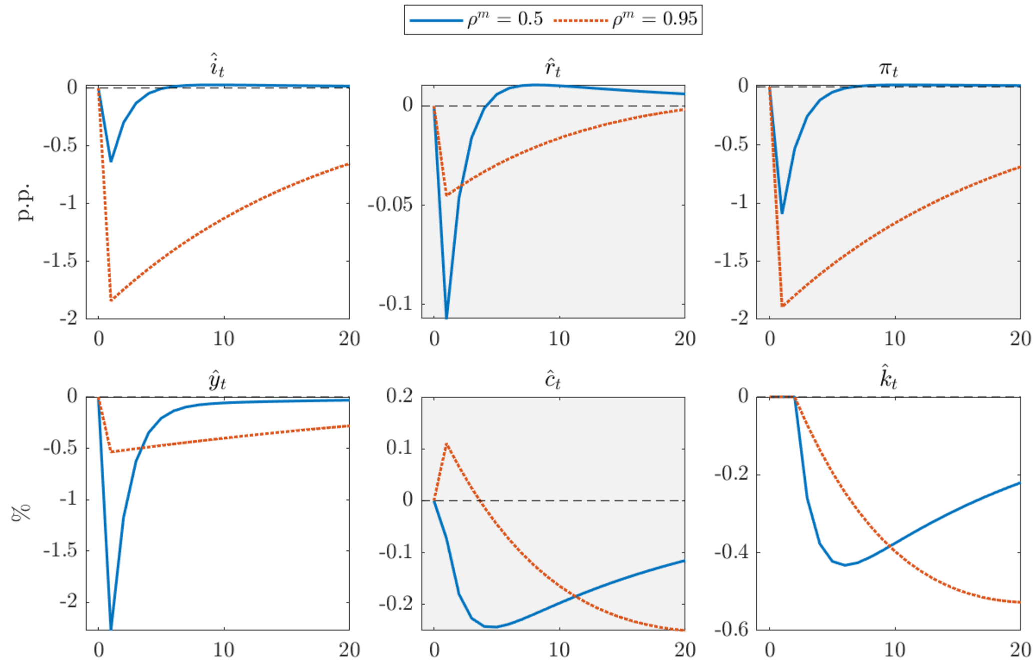


Figure 8: Impulse response function to a one-standard-deviation monetary shock under $\rho^m = 0.5$ and $\rho^m = 0.95$

4 Can Smets and Wouters (2007) do it?

Moving from textbooks to real-world central banking practice, how likely is it that the real interest rate channel identification problem explored in this paper will appear? Quite unlikely. Medium-scale New-Keynesian models often incorporate some additional ingredients that induce consumption smoothing, complementing interest-rate smoothing in the Taylor rule. A more complete specification of the latter — responding to output change, output gap, and inflation expectation — suggests gradualism in monetary policy. Consumption habits, sticky wages, and investment adjustment costs all favor smooth consumption in general.

Taking Smets and Wouters (2007) as a reference and starting from calibration at the mode of the estimated parameters' posterior distribution, I can generate the real interest rate channel identification problem if I reduce the investment adjustment cost parameter (κ^i) from 5.4882 to as low as 0.0001 in addition to setting $\rho^m = 0$ and $\rho^i = 0$ (Figure 9).[11] Since κ^i is a structural parameter, not a policy choice like ρ^m and ρ^i, the negative association between changes in the real interest rate and changes in inflation is arguably much more

[11] At that calibration, the sign switching requires higher values of κ^i. I thank Johannes Pfeifer for providing Dynare codes for the replication of Smets and Wouters (2007).

robust than that between the latter and changes in the nominal interest rate, as it is immune to the Lucas (1976) critique. Figure 10 shows that, immediately after the same monetary shock, just by changing the policy parameter ρ^m when $\rho^i = 0$, it is possible to make the nominal interest rate move in either the same or the opposite direction as inflation.

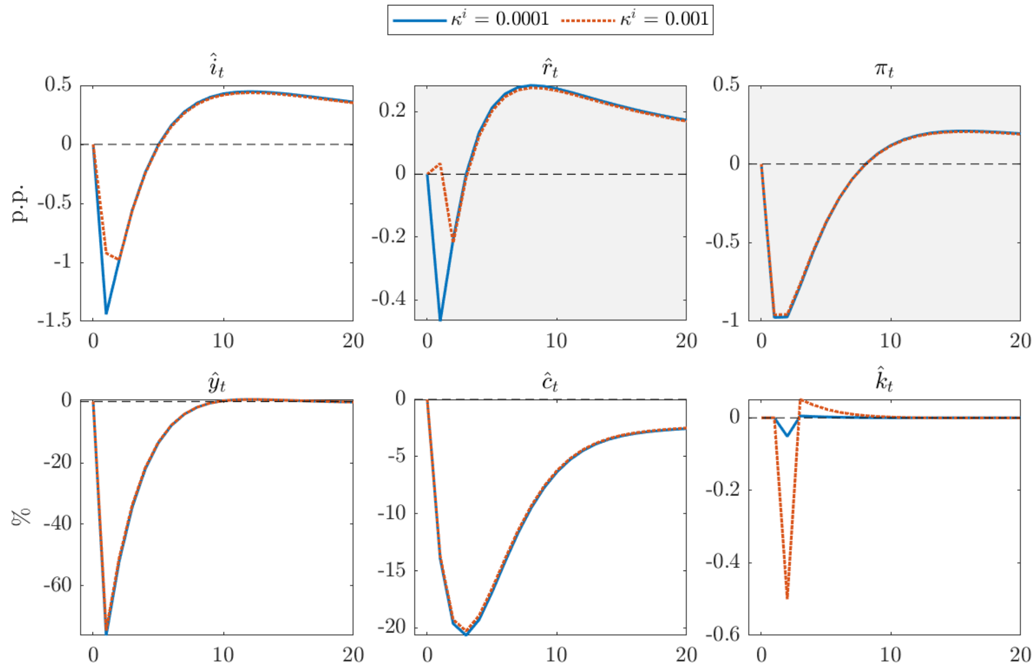


Figure 9: Smets and Wouters (2007)'s impulse response function to a one-standard-deviation monetary shock under $\kappa^i = 0.0001$ and $\kappa^i = 0.001$. Both calibrations assume $\rho^m = 0$ and $\rho^i = 0$.

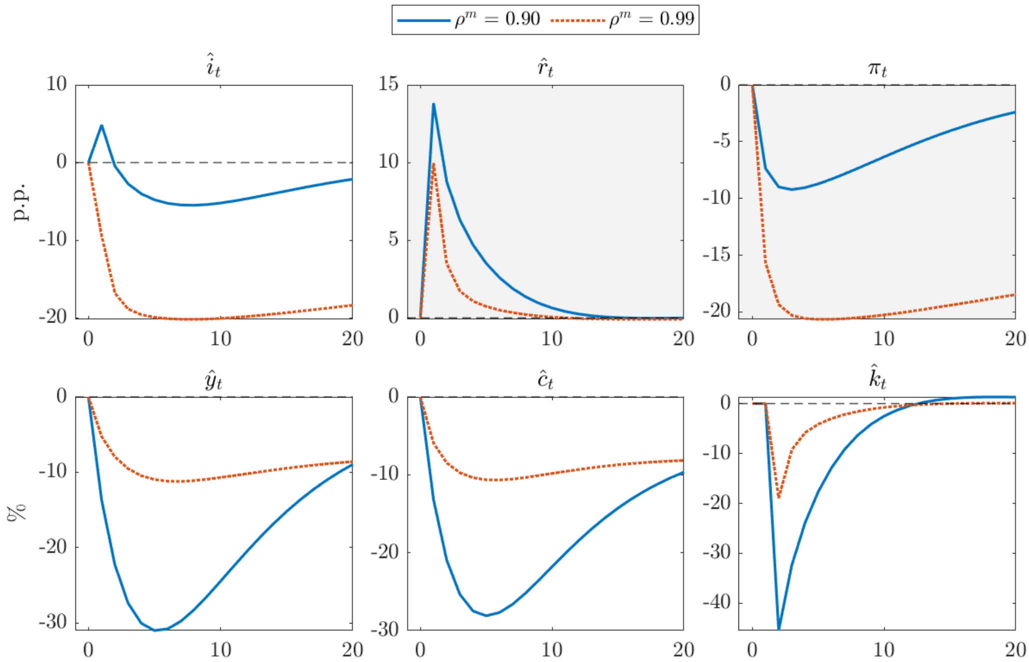


Figure 10: Smets and Wouters (2007)'s impulse response function to a one-standard-deviation monetary shock under $\rho^m = 0.90$ and $\rho^m = 0.99$. Both calibrations assume $\rho^i = 0$.

To show how interest-rate smoothing also helps with the identification problem in the Smets and Wouters (2007) model, I raise the investment adjustment cost parameter from 0.001 to 0.005 in Table 4 and sweep for different values of ρ^m and ρ^i. Note that increasing interest-rate smoothing makes that channel more likely, just like in the textbook model. Finally, in Table 5, I double the investment adjustment costs parameter and reestablish the real interest rate channel.

Table 4: Smets and Wouters (2007)'s parameter sweep with $\kappa^i = 0.005$

	$\rho^i = 0$	$\rho^i = 0.1$	$\rho^i = 0.2$	$\rho^i = 0.3$	$\rho^i = 0.4$	$\rho^i = 0.5$	$\rho^i = 0.6$	$\rho^i = 0.7$	$\rho^i = 0.8$	$\rho^i = 0.9$	$\rho^i = 0.95$	$\rho^i = 0.99$
$\rho^m = 0$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.1$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.2$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.3$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.4$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.5$	+	+	+	-	-	-	-	-	-	+	+	+
$\rho^m = 0.6$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m = 0.7$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m = 0.8$	-	-	-	-	-	-	-	-	-	+	+	+
$\rho^m = 0.9$	-	-	-	-	-	-	+	+	+	+	+	+
$\rho^m = 0.95$	-	-	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.99$	+	+	+	+	+	+	+	+	+	+	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases.

Table 5: Smets and Wouters (2007)'s parameter sweep with $\kappa^i = 0.01$

	$\rho^i = 0$	$\rho^i = 0.1$	$\rho^i = 0.2$	$\rho^i = 0.3$	$\rho^i = 0.4$	$\rho^i = 0.5$	$\rho^i = 0.6$	$\rho^i = 0.7$	$\rho^i = 0.8$	$\rho^i = 0.9$	$\rho^i = 0.95$	$\rho^i = 0.99$
$\rho^m = 0$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.1$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.2$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.3$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.4$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.5$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.6$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.7$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.8$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.9$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.95$	+	+	+	+	+	+	+	+	+	+	+	+
$\rho^m = 0.99$	+	+	+	+	+	+	+	+	+	+	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases.

5 Conclusion

This paper demonstrates that the identification problem of canonical New-Keynesian models augmented with endogenous capital, as revealed by Rupert and Šustek (2019), can be circumvented by including empirically validated interest-rate smoothing, a feature as prevalent in medium-scale New-Keynesian models such as Smets and Wouters (2003, 2007) as capital itself, in the Taylor rule. The sign of changes in the real interest rate right after a positive monetary shock is positive under realistic parameters, thereby reestablishing the observational consistency of the real interest rate channel of monetary policy transmission and weakening the empirical relevance of the New-Keynesian capital puzzle.

This finding suggests that it is acceptable to identify VAR models by imposing the same sign restriction on the real interest rate's response to a monetary shock. It is also acceptable to sequentially order nominal and real rates, as done in Cholesky decompositions. Moreover, using real rates instead of nominal ones still — at least partially — captures monetary policy shocks. Finally, interpretation of impulse responses from canonical DSGE models through the real interest rate channel remains consistent as long as the monetary policy rule includes smoothing.

Acknowledging that at least some smoothing is the norm in central banking and that capital adjustment costs are never negligible in the real world, I conclude that the negative association between changes in inflation and changes in the real interest rate in New-Keynesian models is actually more robust than that between the former and changes in the nominal interest rate, which can be ambiguous depending on how strongly inflation expectations react to monetary shocks.

References

Adjemian, Stéphane, Michel Juillard, Fréderic Karamé, Willi Mutschler, Johannes Pfeifer, Marco Ratto, Normann Rion, and Sébastien Villemot. 2024. "Dynare: Reference Manual, Version 6." CEPREMAP Dynare Working Papers 80.

Altig, David, Lawrence J Christiano, Martin Eichenbaum, and Jesper Lindé. 2011. "Firm-specific capital, nominal rigidities and the business cycle." *Review of Economic Dynamics*, 14(2): 225–247.

Benoit. 1924. "Note Sur Une Méthode de Résolution des équations Normales Provenant de L'Application de la MéThode des Moindres Carrés a un Système D'équations Linéaires en Nombre Inférieur a Celui des Inconnues. — Application de la Méthode a la Résolution D'un Système Defini D'éQuations LinéAires." *Bulletin géodésique*, 2(1): 67–77.

Brault, Joshua, and Hashmat Khan. 2022. "The Real Interest Rate Channel Is Structural in Contemporary New-Keynesian Models: A Note." *Journal of Money, Credit and Banking,* 54(5): 1551–1563.

Carrillo, Julio, Patrick Fève, and Julien Matheron. 2018. "Monetary policy inertia or persistent shocks: a DSGE analysis." *Eighth issue (March 2007) of the International Journal of Central Banking.*

Clarida, Richard, Jordi Galí, and Mark Gertler. 1999. "The science of monetary policy: a new Keynesian perspective." *Journal of Economic Literature,* 37(4): 1661–1707.

Coibion, Olivier, and Yuriy Gorodnichenko. 2012. "Why are target interest rate changes so persistent?" *American Economic Journal: Macroeconomics,* 4(4): 126–162.

Galí, Jordi. 2015. *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications.* . 2nd ed., Princeton, USA:Princeton University Press.

Goodfriend, Marvin. 1987. "Interest rate smoothing and price level trend-stationarity." *Journal of Monetary Economics,* 19(3): 335–348.

Holden, Tom D. 2024. "Robust Real Rate Rules." *Econometrica,* 92(5): 1521–1551.

Ireland, Peter N. 2010. "Monetary transmission mechanism." In *Monetary economics.* 216–223. Springer.

Kaplan, Greg, Benjamin Moll, and Giovanni L Violante. 2018. "Monetary policy according to HANK." *American Economic Review,* 108(3): 697–743.

Kimball, Miles. 1995. "The Quantitative Analytics of the Basic Neomonetarist Model." *Journal of Money, Credit and Banking,* 27(4): 1241–1277.

Lucas, Robert. 1976. "Econometric policy evaluation: A critique." Vol. 1 of *Carnegie-Rochester conference series on public policy,* 19–46, North-Holland.

Rudebusch, Glenn D. 2006. "Monetary Policy Inertia: Fact or Fiction?" *International Journal of Central Banking.*

Rupert, Peter, and Roman Šustek. 2019. "On the mechanics of New-Keynesian models." *Journal of Monetary Economics*, 102: 53–69.

Sack, Brian, and Volker Wieland. 2000. "Interest-rate smoothing and optimal monetary policy: a review of recent empirical evidence." *Journal of Economics and Business*, 52(1-2): 205–228.

Smets, Frank, and Rafael Wouters. 2003. "An estimated dynamic stochastic general equilibrium model of the Euro Area." *Journal of the European Economic Association*, 1(5): 1123–1175.

Smets, Frank, and Rafael Wouters. 2007. "Shocks and frictions in US business cycles: A Bayesian DSGE approach." *American Economic Review*, 97(3): 586–606.

Uhlig, Harald. 2005. "What are the effects of monetary policy on output? Results from an agnostic identification procedure." *Journal of Monetary Economics*, 52(2): 381–419.

Woodford, Michael. 2003*a*. *Interest and Prices: Foundations of a Theory of Monetary Policy.* . 1st ed., Princeton, USA:Princeton University Press.

Woodford, Michael. 2003*b*. "Optimal interest-rate smoothing." *The Review of Economic Studies*, 70(4): 861–886.

A Steady state

In this section, I derive the nonstochastic steady state of the canonical New-Keynesian model augmented with endogenous capital.[12] From their definitions, capital gain and Tobin's q equal 1 at the steady state.

$$\overline{G} = 1 \tag{29}$$

$$\overline{Q} = 1 \tag{30}$$

I pick a zero-inflation steady state. The real rate of return on the implicit risk-free bond of the model is the one obtained from the Fisher equation. Through the no-arbitrage condition,

[12]The steady state is the same with and without interest-rate smoothing.

I combine the two Euler equations of the model, (3) and (19), to get a relation between the two real rates.

$$\overline{\pi} = 0 \tag{31}$$

$$\overline{i} = \overline{R} + \overline{\pi} \quad \Rightarrow \quad \overline{i} = \overline{R} \tag{32}$$

$$\left(1+\overline{i}\right) = \left(1+\overline{\pi}\right)\left(1+\frac{\overline{r}-\delta}{\overline{Q}}\right) \quad \Rightarrow \quad \overline{i} = \overline{r}-\delta \quad \Rightarrow \quad \overline{R} = \overline{r}-\delta \tag{33}$$

From the capital Euler equation (19) evaluated at the steady state I can isolate $\overline{r}$ as a function of the deep parameters:

$$\frac{1}{\overline{C}} = \beta\left(\frac{1}{\overline{C}}\left(\frac{\overline{r}-\delta}{\overline{Q}}+\frac{\overline{Q}}{\overline{Q}}\right)\right) \quad \Rightarrow \quad \overline{r} = \frac{1}{\beta}+\delta-1 \tag{34}$$

Substituting $\overline{r}$ into the FOC of capital (21) at the steady state I get $\frac{\overline{K}}{\overline{L}}$:

$$\overline{r} = \alpha\overline{K}^{\alpha-1}\overline{L}^{1-\alpha} \quad \Rightarrow \quad \frac{\overline{K}}{\overline{L}} = \left(\frac{\alpha}{\frac{1}{\beta}-1+\delta}\right)^{\frac{1}{1-\alpha}} \tag{35}$$

The production function (20) at the steady state can be rewritten so as to put in evidence $\frac{\overline{K}}{\overline{L}}$ on the right side of the expression:

$$\overline{Y} = \left(\frac{\overline{K}}{\overline{L}}\right)^{\alpha}\overline{L} \tag{36}$$

Investment at the steady state only compensates for depreciated capital.

$$\overline{I} = \delta\overline{K} \tag{37}$$

Now, the aggregate resource constraint (23) may be rewritten as auxiliary ratios by substituting the just derived expressions for $\overline{Y}$ and $\overline{I}$ and then dividing by $\overline{L}$.

$$\overline{Y} = \overline{C}+\overline{I} \quad \Rightarrow \quad \left(\frac{\overline{K}}{\overline{L}}\right)^{\alpha} = \frac{\overline{C}}{\overline{L}}+\frac{\delta\overline{K}}{\overline{L}} \tag{38}$$

Combining the condition for the optimal mix of capital and labor in production (21) and the intratemporal condition at the steady state (4), I can write an expression for $\frac{\overline{C}}{\overline{L}}$:

$$\overline{W} = (1-\alpha)\,\overline{K}^{\alpha}\overline{L}^{-\alpha} \quad \text{and} \quad \frac{\overline{W}}{\overline{C}} = \overline{L}^{\eta} \quad \Rightarrow \quad \frac{\overline{C}}{\overline{L}} = (1-\alpha)\left(\frac{\overline{K}}{\overline{L}}\right)^{\alpha}\overline{L}^{-(\eta+1)} \tag{39}$$

By substituting previously derived expressions for $\frac{\overline{K}}{\overline{L}}$ and $\frac{\overline{C}}{\overline{L}}$ into the aggregate resource constraint, I obtain $\overline{L}$ as a function of the deep parameters of the model.

$$\overline{L} = \left(\left(\frac{1}{1-\alpha}\right)\left(\frac{\delta\alpha}{\frac{1}{\beta}-1+\delta}\right)\right)^{\frac{-1}{\eta+1}} \tag{40}$$

I get the remaining steady-state variables as functions of the parameters by recursively substituting (40) into my previously derived expressions: (35) $\Rightarrow$ $\overline{K}$, (36) $\Rightarrow$ $\overline{Y}$, (37) $\Rightarrow$ $\overline{I}$, and (39) $\Rightarrow$ $\overline{C}$ and $\overline{W}$. Finally, I obtain $\overline{\chi}$ by substituting $\overline{r}$ and $\overline{W}$ into (22) evaluated at the steady state.